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THE
COMPENDIOUS MEASURER;

BEING A BRIEF, YET COMPREHENSIVE,

TREATISE ON MENSURATION,

AND

PRACTICAL GEOMETRY.

WITH

AN INTRODUCTION TO DECIMAL AND DUODECIMAL

ARITHMETIC.

ADAPTED TO PRACTICE, AND THE USE OF SCHOOLS.

By CHARLES HUTTON, LL.D. AND F.R.S. &c.

— *K* —
THE FOURTH EDITION, CORRECTED AND ENLARGED;

ILLUSTRATED WITH THE PLAN OF A FIELD-BOOK ENGRAVED ON
COPPER.

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P R E F A C E.

SOME years since I published a complete Treatise on Mensuration, both in Theory and Practice; in which the Elements of that Science are demonstrated, and the Rules applied to the various practical purposes of life. That work has been well received by the Public, and honoured with the high approbation of the more learned Mathematicians.

It has, however, been often represented to me, by Tutors and others, that the great size and price of that work, as well as the very scientific manner in which it is formed, prevent it from being so generally useful in schools, and to practical measurers, as a more compendious and familiar little book might be, which they could put into the hands of their pupils, as a work containing all the practical rules of that art, in a form proper for them to copy from, and unmixed with such geometrical and algebraical demonstrations as occur in the large work.

In compliance, therefore, with such representations, I have drawn up this Compendium of Mensuration, Practical Geometry, and Arithmetic, expressly

with the view of accommodating it to practical matters, and to the use of schools. I have, therefore, here brought together all the most useful rules and precepts; have arranged them in an orderly manner, proper for the pupil to copy; and delivered them in plain and familiar language. An example, worked out at full length, is set down to each rule, together with drawings or representations of the geometrical figures proper to illustrate each problem; and then are subjoined some more questions to each rule, as examples proposed for the practice of the learner, with the answer set down, by which he may know when his work is right.

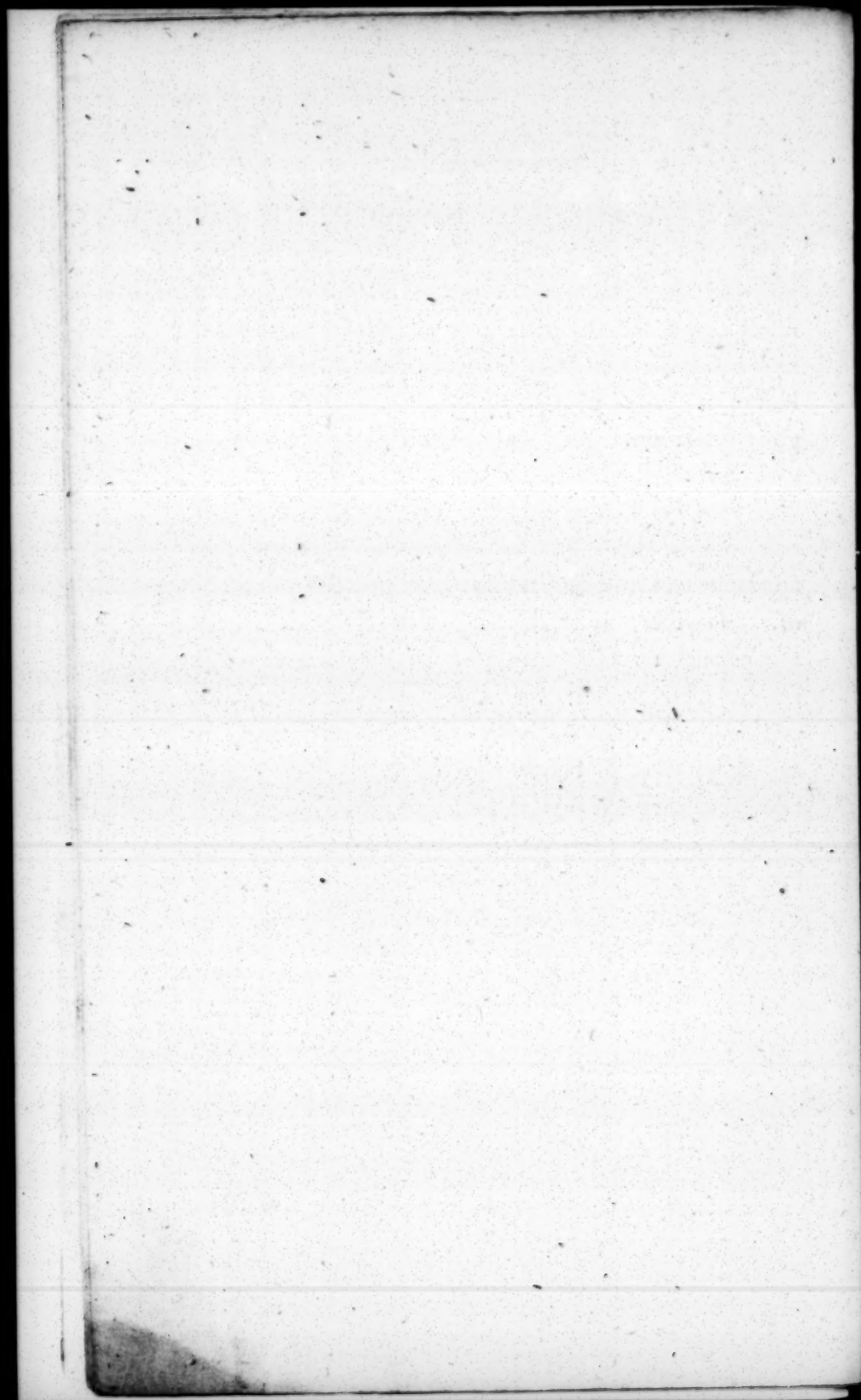
The Introduction to Decimal and Duodecimal Arithmetic will be found useful, by going over those branches before entering on the Mensuration, that the learner may be very ready and expert in numeral calculations.

The Practical Geometry contains a great number of geometrical constructions and operations; by the practice of which the learner will acquire the free and easy use of his instruments, and so become prepared for making the drawings that are useful for illustrating the various branches of Mensuration following.

The Mensuration itself next succeeds, and is divided into various parts; first, Mensuration in general, and then as applied to the several practical uses
in

in life. The whole being arranged in such order as the learner may properly take in succession; or distinguished into several branches, of which he may select and study any peculiar ones that may be more to his purpose than the rest, when he has not either leisure or inducement to go over the whole in a regular gradation. And notwithstanding the compendious size of the book, and the great number of practical branches here treated, it will be found that each one is much more full and complete than the first appearance of so small a form may promise to admit of. However, if farther satisfaction be desired by any one, either concerning the science in general, or the demonstrations of the rules, or the more curious and copious display of properties, he may apply to my large treatise before mentioned, where he will find every part delivered in the most ample form.

To this edition is added the new method of surveying, now practised by the best surveyors, illustrated with a map or plan, and an engraved form of the Field Book.



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INTRODUCTION.

DECIMAL FRACTIONS.

A DECIMAL is a fraction whose denominator is an unit, or 1, with some number of ciphers annexed; as $\frac{1}{10}$ or $\frac{45}{1000}$.

Decimals are written down without their denominators, the numerators being so distinguished as to shew what the denominators are; which is done, by separating, by a point, so many of the right-hand figures from the rest as there are ciphers in the denominator; the figures on the left side of the point being integers, and those on the right decimals.

Thus, 0.5 is understood to be $\frac{5}{10}$ or $\frac{1}{2}$

And 0.25 is - - $\frac{25}{100}$ or $\frac{1}{4}$

And 0.75 is - - $\frac{75}{100}$ or $\frac{3}{4}$

And 1.3 is - - $\frac{13}{10}$ or $1\frac{3}{10}$

And 24.6 is - - $24\frac{6}{10}$ or $24\frac{3}{5}$

But when there is not a sufficient number of figures in the numerator, ciphers are prefixed to supply the defect.

B

So,

Ciphers on the right of decimals do not alter their value.

For .5 or $\frac{5}{10}$ that is $\frac{1}{2}$
 And .50 or $\frac{50}{100}$ that is $\frac{1}{2}$
 And .500 or $\frac{500}{1000}$ that is $\frac{1}{2}$

&c, are all of equal value.

But ciphers before decimal figures, and after the separating point, diminish their value in a ten-fold proportion for every cipher.

So, .5 is $\frac{5}{10}$ or $\frac{1}{2}$
 But .05 is $\frac{5}{100}$ or $\frac{1}{20}$
 And .005 is $\frac{5}{1000}$ or $\frac{1}{200}$

And so on.

So that, in any mixed or fractional number, if the separating point be moved

one, two, three, &c, places to the right-hand, every figure will be

10, 100, 1000, &c, times greater than before.

But if the point be moved towards the left hand; then every figure will be diminished in the same manner, or the whole quantity will be divided by

10, 100, 1000, &c.

INTRODUCTION.

ADDITION OF DECIMALS.

SET the numbers under each other according to the value of their places, as in whole numbers, or so that the decimal points stand directly below each other. Then add as in whole numbers, placing the decimal point in the sum straight below the other points.

EXAMPLES.

(1)	(2)	(3)
276	7530	312.09
39.213	16.201	6.5711
72014.9	3.0142	4195.6
417.	957.13	71.498
5032.	6.72819	9739.215
2214.298	.03014	179.
79993.411	8513.10353	14500.9741

Ex. 4. What is the sum of .014, .9816, .32, .15914, 72913, and .0047?

Ex. 5. What is the sum of 27.148, 918.73, 14016, 294304, .7138, and 221.7?

Ex. 6. Required the sum of 312.984, 21.3918, 2700.42, 3.153, 27.2, and 581.06.

SUB-

SUBTRACTION OF DECIMALS.

SET the less number under the greater in the same manner as in addition. Then subtract as in whole numbers, and place the decimal point in the remainder straight below the other points.

	(1)	(2)	(3)
From	9173	273	21481
take	2138	19185	490142
	<hr/>	<hr/>	<hr/>
rem.	7035	08115	20990858
	<hr/>	<hr/>	<hr/>

Ex. 4. What is the difference between 91713 and 407?

Ex. 5. What is the difference between 2714 and 916?

Ex. 6. What is the difference between 1637 and 800135?



MULTIPLICATION OF DECIMALS.

SET down the factors under each other, and multiply them as in whole numbers, then from the product, towards the right hand, point off as many figures for decimals, as there are decimal places in both factors together.

But if there be not as many figures in the product as there ought to be decimals, prefix the proper number of ciphers to supply the defect.

EXAMPLES.

(1)	(2)	(3)
520.3	91.78	.217
.417	.381	.0431
<hr/>	<hr/>	<hr/>
36421	9178	217
5203	73424	651
20812	27534	868
<hr/>	<hr/>	<hr/>
216.9651	34.96818	.0093527
<hr/>	<hr/>	<hr/>

Ex. 4. What is the product of 51.6 and 21?

Ex. 5. What is the product of 314 and .029?

Ex. 6. What is the product of .051 and .0091?

Note, When decimals are to be multiplied by 10, or 100, or 1000, &c, that is by 1 with any number of ciphers, it is done by only moving the decimal point as many places farther to the right hand, as there are ciphers in the said multiplier: subjoining ciphers if there be not so many figures.

EXAMPLES.

1. The product of 51.3 and 10 is 513
2. The product of 2.714 and 100 is
3. The product of .9163 and 1000 is
4. The product of 21.31 and 10000 is

CONTRACTION.

When the product would contain several more decimals than are necessary for the purpose in hand, the work may be much contracted thus, retaining only the proper number of decimals.

Set

Set the units figure of the multiplier straight under such decimal place of the multiplicand as you intend the last of your product shall be, writing the other figures of the multiplier in an inverted order: then in multiplying, reject all the figures in the multiplicand which are on the right of the figure you are multiplying by; setting the products down so, that their right-hand figures fall straight below each other; and carrying to such right-hand figures from the product of the two preceding figures in the multiplicand thus, viz. 1 from 5 to 14, 2 from 15 to 24, 3 from 25 to 34, &c, inclusively; and the sum of the lines will be the product to the number of decimals required, and will commonly be to the nearest unit in the last figure.

EXAMPLES.

1. Multiply 27.14986 by 92.41035 , so as to retain only four places of decimals in the product.

Contracted.

27.14986
 53014.29

24434874
 542997
 108599
 2715
 81
 14

2508.9280

Common way.

27.14986
 92.41035

$13|574930$
 $81|44958$
 $2714|986$
 $108599|44$
 $542997|2$
 $24434874|$

$2508.9280|650510$

2. Multiply 480.14936 by 2.72416 , retaining four decimals in the product.

3. Multiply 2490.3048 by $.573286$, retaining five decimals in the product.

4. Multiply 325.701428 by $.7218393$, retaining three decimals in the product.

DIVISION OF DECIMALS.

DIVIDE as in whole numbers. And to know how many decimals to point off in the quotient, observe the following rules:

1. There must be as many decimals in the dividend, as in both the divisor and quotient together; therefore, point off for decimals in the quotient, as many figures, as the decimal places in the dividend exceed those in the divisor.

2. If the figures in the quotient are not so many as the rule requires, supply the defect by prefixing ciphers.

3. If the decimal places in the divisor be more than those in the dividend, add ciphers as decimals to the dividend, till the number of decimals in the dividend be equal to those in the divisor, and the quotient will be integers till all these decimals are used. And in case of a remainder after all the figures of the dividend are used, and more figures are wanted in the quotient, annex ciphers to the remainder, to continue the division as far as necessary.

4. The first figure of the quotient will possess the same place, of integers or decimals, as that figure of the dividend which stands over the units place of the first product.

EXAMPLES.

1. Divide 3424.6056 by 43.6. 2. Divide 3877875 by .675.

$$\begin{array}{r}
 43 \cdot 6 \overline{) 3424 \cdot 6056 (78 \cdot 546} \\
 \underline{3726} \\
 2380 \\
 \underline{2005} \\
 2616
 \end{array}
 \qquad
 \begin{array}{r}
 \cdot 675 \overline{) 3877875 \cdot 000 (5745000} \\
 \underline{5028} \\
 3037 \\
 \underline{3375} \\
 \dots 000
 \end{array}$$

3. Divide

3. Divide $\cdot 0081892$ by $\cdot 347$ 5. Divide $3\cdot 15$ by 375 .
 4. Divide $7\cdot 13$ by $\cdot 18$. 6. Divide 109 by $\cdot 215$.

CONTRACTIONS.

1. If the divisor be an integer with any number of ciphers at the end; cut them off, and remove the decimal point in the dividend so many places farther to the left as there were ciphers cut off, prefixing ciphers if need be; then proceed as before.

EXAMPLES.

1. Divide 953 by 21000 . 2. Divide 41020 by 32000 .

$$\begin{array}{r} 21\cdot 000 \overline{) 953} \\ 7 \overline{) 31766} \\ \cdot 04538 \text{ \&c.} \end{array}$$

$$\begin{array}{r} 32\cdot 000 \overline{) 41\cdot 020} \\ 8 \overline{) 10\cdot 255} \\ 1\cdot 281875 \end{array}$$

Here I first divide by 3, and then by 7, because 3 times 7 is 21.

Here I first divide by 4, and then by 8, because 4 times 8 is 32.

3. Divide $45\cdot 5$ by 2170 . 4. Divide 61 by 79000 .

2. Whence, if the divisor be 1 with ciphers, the quotient will be the same figures with the dividend, having the decimal point so many places farther to the left as there are ciphers in the divisor.

EXAMPLES.

$$\begin{array}{l} 2173 \div 100 = 21\cdot 73 \\ 5\cdot 16 \text{ by } 1000 = \end{array}$$

$$\begin{array}{l} 419 \text{ by } 10 = \\ \cdot 21 \text{ by } 1000 = \end{array}$$

3. When the number of figures in the divisor is great, the division at large will be very troublesome, but may be contracted thus:

Having by the 4th general rule, found what place of decimals or integers the first figure of the quotient

B 5

will

will possess; consider how many figures of the quotient will serve the present purpose; then take the same number of the left-hand figures of the divisor, and as many of the dividend figures as will contain them (less than 10 times); by these find the first figure of the quotient; and for each following figure divide the last remainder by the divisor, wanting one figure to the right more than before, but observing what must be carried to the first product for such omitted figures, as in the second contraction of Multiplication; and continue the operation till the divisor is exhausted.

When there are not so many figures in the divisor as are required to be in the quotient, begin the division with all the figures as usual, and continue it till the number of figures in the divisor, and those remaining to be found in the quotient be equal, after which use the contraction.

EXAMPLES.

1. Divide 2508.92806 by 92.41035 , so as to have four decimals in the quotient.—In this case the quotient will contain six figures. Hence

$$\begin{array}{r}
 92.41035 \) \ 2508.928,06 \ (\ 27.1498 \\
 \underline{660721 \cdot} \\
 13849 \cdot \cdot \\
 \underline{4608 \cdot \cdot \cdot} \\
 912 \cdot \cdot \cdot \cdot \\
 \underline{80 \cdot \cdot \cdot \cdot \cdot} \\
 6 \cdot \cdot \cdot \cdot \cdot \cdot
 \end{array}$$

2. Divide 4109.2351 by 230.409 so that the quotient may contain four decimals.

3. Divide 37.10438 by 5713.96 that the quotient may contain five decimals.

4. Divide 913.08 by 2137.2 that the quotient may contain three decimals.

REDUC.

REDUCTION OF DECIMALS.

1. To reduce a vulgar fraction to a decimal. Divide the numerator, with as many decimal ciphers annexed, as may be necessary, by the denominator; and the quotient will be the decimal sought.

EXAMPLES.

1. Reduce $\frac{1}{99}$ to a decimal. 2. Reduce $\frac{1}{75}$ to a decimal

$$\begin{array}{r} 9 \overline{) 1.000000} \\ 11 \overline{) 0.111111} \\ \hline 0.010101 \text{ \&c} = \frac{1}{99} \end{array}$$

$$\begin{array}{r} 5 \overline{) 1.0} \\ 5 \overline{) 0.20} \\ 3 \overline{) 0.04} \\ \hline 0.01333 \text{ \&c} = \frac{1}{75} \end{array}$$

Here divide by 9 and 11, because 9 times 11 is 99. And the decimal value of $\frac{1}{99}$ is the circulate $\cdot\dot{0}1$.

Here divide by 5, 5 and 3, because $5 \times 5 \times 3 = 75$. And the decimal value of $\frac{1}{75}$ is the repetend $\cdot 01\dot{3}$.

OTHER EXAMPLES.

$\frac{1}{2} = .5$	$\frac{1}{3} = .\dot{3}$	$\frac{1}{8} = .125$
$\frac{1}{3} = .\dot{3}$	$\frac{1}{8} = .1\dot{6}$	$\frac{1}{9} = .\dot{1}$
$\frac{1}{4} = .25$	$\frac{1}{7} = .14285\dot{7}$	$\frac{1}{10} = .1$
$\frac{2}{7} = .28571\dot{4}$	$\frac{14}{191} = .07330\dot{3}$	$\frac{1}{753} = .001328\dot{1}$

So that whenever we meet with the repetend $\cdot\dot{3}$, in any operation, we may substitute $\frac{1}{3}$ for it: in like manner we may take $\frac{2}{3}$ for $\cdot\dot{6}$, and $\frac{1}{8}$ for $\cdot 1\dot{6}$, and $\frac{1}{9}$ for $\cdot\dot{1}$, and $\frac{2}{9}$ or 1 for $\cdot\dot{9}$, &c.

Note, When a great many figures are required in the decimal, and the denominator of the given fraction is a prime number greater than 11, the operation will be best performed as follows.

Suppose, for instance, I would find the reciprocal of the prime number 29, or the value of the fraction $\frac{1}{29}$ in decimal numbers. I divide 1.000 by 29, in the common way, so far as to find two or three of the first figures, or till the remainder becomes a single figure, and then I assume the supplement to complete the quotient. Thus I shall have $\frac{1}{29} = 0.03448\frac{8}{29}$ for the complete quotient; which equation if I multiply by the numerator 8, it will give $\frac{8}{29} = 0.27584\frac{64}{29}$ or rather $\frac{8}{29} = 0.27586\frac{6}{29}$. I substitute this instead of the fraction in the first equation, and I shall have $\frac{1}{29} = 0.0344827586\frac{6}{29}$. Again I multiply this equation by 6, and it will give $\frac{6}{29} = 0.2068965517\frac{7}{29}$, and then by substitution $\frac{1}{29} = 0.03448275862068965517\frac{7}{29}$. Again, I multiply this equation by 7, and it becomes $\frac{7}{29} = 0.24137931034482758620\frac{1}{29}$, and then by substitution $\frac{1}{29} = 0.0344827586206896551724137931034482758620\frac{1}{29}$; where every operation will at least double the number of figures found by the preceding operation. And this will be an easy expedient for converting division into multiplication in all cases. For this reciprocal of the divisor being thus found, it may be multiplied by the dividend to produce the quotient.

II. To reduce a decimal to a Vulgar Fraction.

Under the figures of the given decimal write its proper denominator; which fraction, abbreviated as much as it can be, will be the vulgar fraction sought.

EXAMPLES.

$$\begin{aligned} \text{So } .5 &= \frac{5}{10} = \frac{1}{2} \\ \text{And } .25 &= \frac{25}{100} = \frac{1}{4} \\ \text{And } .75 &= \frac{75}{100} = \frac{3}{4} \\ \text{And } .6 &= \frac{6}{10} = \frac{3}{5} \\ \text{And } .625 &= \frac{625}{1000} = \frac{5}{8} \\ \text{And } .5625 &= \frac{5625}{10000} = \frac{9}{16} \end{aligned}$$

III. To find the Value of a decimal, in the Lower Denominations.

Multiply the given decimal by the number of parts in the next lower denomination; from the product cut off as many decimals as are in the given number.

Multiply these by the parts in the next lower denomination again, cutting off the same number of decimals as before.

And proceed in the same manner to the lowest denomination: then the several integer parts cut off on the left hand will give the value of the decimal proposed.

EXAMPLES.

1. For the value of .3914l.

$$\begin{array}{r} .3914 \\ \times 20 \\ \hline 78280 \\ \times 12 \\ \hline 99360 \\ \times 4 \\ \hline 37440 \\ \hline \text{Ans. } 7s \ 9\frac{1}{4}d \end{array}$$

2. For the value of .2139lb. avoir.

$$\begin{array}{r} .2139 \\ \times 16 \\ \hline 12834 \\ \times 16 \\ \hline 34224 \\ \hline \text{Ans. } 3 \text{ oz } 6 \text{ dr} \end{array}$$

OTHER EXAMPLES.

<i>Questions.</i>	<i>Answers.</i>
1. — .775 l	16s 6d
2. — .625 s	0 7½ d
3. — .8635 l	17 3
4. — .0125 lb troy	3 dwts
5. — .4694 lb troy	5 oz 12 dwts 15 gr
6. — .625 cwt	2 qr 14 lb
7. — .009943 mile	17 yd 1 f 6 in almost
8. — .6875 yd cloth	2 qu 3 nl
9. — .3375 acr	1 rd 14 pl
10. — .2083 hhd wine	13 gl
11. — .40625 qr corn	3 bu 1 pk
12. — .42857 month	1 wk 5 day nearly

IV. To bring Quantities to Decimals of Higher Denominations.

CASE I.

If a single integer or decimal be proposed, reduce it to the higher denomination, by dividing as in reduction of whole numbers.

EXAMPLES.

1. Reduce 9d to the decimal of a pound.

$$\begin{array}{r|l}
 12 & 9 \text{ d} \\
 20 & 0.75 \text{ s} \\
 \text{Ans.} & \underline{0.0375 \text{ l}}
 \end{array}$$

2. Reduce 1 dwt to the decimal of a lb.

$$\begin{array}{r|l}
 20 & 1 \text{ dwt} \\
 12 & 0.05 \text{ oz} \\
 \text{Ans.} & \underline{0.00416 \text{ lb}}
 \end{array}$$

Questions.

<i>Questions.</i>	<i>Answers.</i>
3. Reduce .26 d to l sterl - -	.001083 l
4. Reduce 7 drams to lb avoird -	.02734375 lb
5. Reduce 2.15 lb to a cwt - -	.019196 cwt
6. Reduce 24 yds to a mile -	.013636 mile
7. Reduce .056 pole to an acre -	.00035 acre
8. Reduce 1.2 pint to hd wine -	.00238 hd
9. Reduce 14 min to a day -	.009722 day
10. Reduce .21 pint to a peck -	.013125 peck

CASE II.

A compound number may be reduced to a superior name by reducing each of its parts, and taking the sum of the decimals; the best way to do which is thus:

Write the given numbers under each other, proceeding orderly from the least to the greatest name, for dividends; draw a perpendicular line on the left of these, and on the left of it write opposite to each dividend such a number, for a divisor, as will reduce it to the next superior name; then begin with the upper division, and affix the quotient of each to the next dividend, as a decimal part of it, before it be divided, and the last sum will be the answer.

EXAMPLES.

1. Reduce 3l 12s 6 $\frac{1}{4}$ d to the denomination of l. 2. Reduce 5 oz 12 dwt 16 gr to the denom. of lb.

$$\begin{array}{r|l}
 4 & 3 \\
 12 & 6.75 \\
 20 & 12.5625 \\
 \text{Ans.} & .628125
 \end{array}$$

$$\begin{array}{r|l}
 24 & 16 \\
 20 & 12.66\dot{6} \\
 12 & 5.633\dot{3} \\
 \text{Ans.} & 0.4694
 \end{array}$$

Questions.

<i>Questions.</i>	<i>Answers</i>
3. Reduce 19 l 17 s 4½ d to l	19.8635416 l
4. Reduce 15 s 6 d to l	.775 l
5. Reduce 7½ d to a shil	.625 s
6. Reduce 3 cwt 2 qr 14 lb to cwt	3.625 cwt
7. Reduce 17 yd 1 ft 6 in to a mile	.00994318 mil
8. Reduce 2 qr 3 nls to a yard	.6875
9. Reduce 13 ac 1 r 14 pol to acres	13.3375 acr
10. Reduce 13 gal 1 pint to hd wine	.2083 hd
11. Reduce 3 bush 1 pec to a qr	.40625 qr
12. Reduce 3 mo 1 we 5 da to mon	3.42857 mon

CIRCULATING DECIMALS.

It has already been observed, that when an infinite decimal repeats always one figure, it is a single repetend; and when more than one, a compound repetend, or a circulate: also that a point is set over a single repetend, and a point over the first and last figures of a circulate.

It may farther be observed, that when other decimal figures precede a repetend, in any numbers, it is called a mixed number or quantity, as $\cdot\dot{2}3$, or $\cdot104\dot{1}23$: otherwise it is a pure repetend, as $\cdot\dot{3}$ and $\cdot\dot{1}23$.

Similar repetends begin at the same place, and consist of the same number of figures: as $\cdot\dot{3}$ and $\cdot\dot{6}$, or $1\cdot\dot{3}4\dot{1}$ and $2\cdot\dot{1}5\dot{6}$.

Diffimilar repetends begin at different places, and consist of an unequal number of figures.

Similar and conterminous repetends begin and end at the same place, as $2\cdot9\dot{1}0\dot{4}$ and $\cdot0\dot{6}\dot{1}3$.

REDUC-

REDUCTION OF REPEATING DECIMALS.

CASE I.

To reduce a Single Repetend to a Vulgar Fraction.

Make the given decimal the numerator; and for a denominator take as many nines as there are recurring places in the given repetend.

If one or more of the left-hand places, in the given decimal, be ciphers, annex as many ciphers to the right-hand of the nines in the denominator.

EXAMPLES.

- 1 So $\dot{3} = \frac{3}{9} = \frac{1}{3}$. And $2.\dot{6}3 = 2\frac{63}{99} = 2\frac{7}{11}$.
- 2 And $\dot{0}5 = \frac{5}{90} = \frac{1}{18}$. 5 And $\dot{0}59440\dot{5} = \frac{594405}{9999999} = \frac{17}{18}$.
- 3 And $\dot{1}23 = \frac{123}{999} = \frac{41}{333}$. 6 And $\dot{7}69230 = \frac{769230}{999999} = \frac{19}{18}$.

CASE II.

To reduce a mixed repetend to a Vulgar Fraction.

To as many nines as there are figures in the repetend, annex as many ciphers as there are finite places, for the denominator of the vulgar fraction.

Multiply the nines in the denominator by the finite part of the decimal, and to the product add the repeating part, for the numerator.

Or find the vulgar fraction, as before, answering to the repetend then join it to the finite part, and reduce them to a common denominator.

EXAM-

EXAMPLES.

1. So $\cdot 5\dot{3} = \frac{9 \times 5 + 3}{90} = \frac{48}{90} = \frac{8}{15}$
2. And $\cdot 58\dot{3} = \frac{58 \times 9 + 3}{900} = \frac{525}{900} = \frac{7}{12}$
3. And $\cdot 13\dot{8} = \frac{13 \times 9 + 8}{900} = \frac{125}{900} = \frac{5}{36}$
4. And $\cdot 592\dot{5} = \frac{5 \times 999 + 925}{9990} = \frac{5920}{9990} = \frac{16}{27}$

ADDITION OF REPETENDS.

MAKE every line to begin and end at the same place, by extending the repetends, and filling up the vacancies with the proper figures and ciphers. Then add as in common numbers; only increase the sum of the right hand row, or last row of the repetends, by as many units as the first row of repetends contains nines. And the sum will circulate at the same places as the other lines.

EXAMPLES.

(1)	(2)
$39\cdot 654\dot{8} = 39\cdot 6548\dot{0}$	$91\cdot 357 = 91\cdot 357\dot{0}$
$81\cdot 04\dot{6} = 81\cdot 0466\dot{6}$	$72\cdot 38 = 72\cdot 388\dot{8}$
$42\cdot 3\dot{5} = 42\cdot 3555\dot{5}$	$7\cdot 21 = 7\cdot 211\dot{1}$
$9\cdot 837 = 9\cdot 8377\dot{7}$	$4\cdot 296\dot{5} = 4\cdot 2965\dot{5}$
<u>Sum</u> $172\cdot 8948\dot{0}$	<u>Sum</u> $175\cdot 2535\dot{5}$

(3)

(3)	(4)
$9\dot{8}1\dot{4} = 9\cdot814\dot{8}148\dot{1}$	$2\cdot4\dot{1} = 2\cdot4\dot{1}$
$1\cdot5 = 1\cdot50000000$	$13\cdot21\dot{5} = 13\cdot21\dot{5}151\dot{5}1$
$87\cdot2\dot{6} = 87\cdot2666666\dot{6}$	$5\cdot8 = 5\cdot8$
$0\cdot8\dot{3} = 0\cdot8333333\dot{3}$	$27\cdot09\dot{6} = 27\cdot096969\dot{6}$
$125\cdot09 = 125\cdot0909090\dot{9}$	$0\cdot91\dot{3} = 0\cdot9139139\dot{1}$
<u>Sum 223·50572390</u>	<u>Sum 49·43603512</u>

SUBTRACTION OF REPETENDS.

MAKE the repetends to begin and end together, as in addition. Then subtract as usual; only, if the repetend of the number to be subtracted exceed the repetend of the other number, make the last figure of the remainder 1 less than it otherwise would be.

EXAMPLES.

(1)	(2)
$76\cdot3\dot{2} = 76\cdot322\dot{2}$	$89\cdot57\dot{6} = 89\cdot576\dot{0}$
$54\cdot761\dot{7} = 54\cdot7617\dot{7}$	$12\cdot584\dot{6} = 12\cdot5846\dot{6}$
Diff. <u>21·5604</u>	Diff. <u>76·9913</u>
(3)	(4)
$29\cdot2\dot{1} = 29\cdot2121\dot{2}1$	$87\cdot416\dot{1} = 87\cdot4161\dot{4}$
$3\cdot561\dot{7} = 3\cdot561561\dot{7}$	$0\cdot53\dot{2} = 0\cdot5323\dot{2}$
Diff. <u>25·650559</u>	Diff. <u>86·88381</u>

MUL.

MULTIPLICATION OF REPETENDS.

1. WHEN a repetend is to be multiplied by a finite number: Multiply as in common numbers; only observe what must be carried from the beginning of the repetend to the end of it. And make all the lines begin and end together when they are to be added.

2. In multiplying a finite decimal by a single repetend; multiply by the repetend, and divide by 9 or $\frac{9}{10}$.

3. In more complex cases, reduce the repetends to vulgar fractions; then divide these, and reduce the quotient to a decimal, if necessary.

EXAMPLES.

(1)	(2)	(3)
716 [.] 2935 [.]	2 [.] 104 [.]	3 [.] 028 [.]
27	12	17
50140548 [.]	4208 [.]	21202 [.]
143258711 [.]	21044 [.]	30288 [.]
193399260 [.]	25253 [.]	51491 [.]

(4)

$$\begin{array}{r} 27 \cdot 1241 \\ \quad 3 \cdot 6 \\ \hline 9 \mid 1627446 \\ \hline 1808273 \\ 813723 \\ \hline 9945503 \end{array}$$

Or $3 \cdot 6 = 3\frac{6}{10} = 3\frac{3}{5} = \frac{11}{5}$.

Then $27 \cdot 1241$

$$\begin{array}{r} 3 \mid 2983651 \\ \hline 9945503 \end{array}$$

(5)

$$\begin{aligned} & \text{(5)} \\ & \text{Mult. } 1'20\dot{6} \text{ by } 3'5 \\ & 1'20\dot{6} = 1'2\frac{0}{99} = 1'2\frac{2}{33} = 1\frac{68}{330} = \frac{398}{330} \\ & \text{and } 3'5 = 3\frac{5}{9} = \frac{32}{9} ; \\ & \text{then } \frac{398}{330} \times \frac{32}{9} = \frac{12736}{2970} = 4'2882154 \end{aligned}$$

DIVISION OF REPETENDS.

1. If the dividend only be a repetend, divide as in common numbers, bringing down always the recurring figures, till the quotient become as exact as requisite.

2. And if the divisor only be a repetend, it will be best to change it into its equivalent vulgar fraction, then multiply by its numerator, and divide by its denominator.

3. But if both divisor and dividend be repetends, change them both to vulgar fractions.

EXAMPLES.

$$\begin{array}{r} \text{(1)} \\ 1'2 \overline{) 2'525\dot{3}} \\ \underline{2'104} \end{array}$$

$$\begin{array}{r} \text{(3)} \\ 17 \overline{) 51'491} \text{ (} 3'028 \\ \underline{49} \\ 151 \\ \underline{151} \end{array}$$

$$\begin{array}{r} \text{(2)} \\ 8 \overline{) 27'91\dot{2}} \\ \underline{3'489027} \end{array}$$

$$\begin{array}{r} \text{(4)} \\ 27 \overline{) 193'39926} \\ 9 \overline{) 64'46642} \\ \underline{7'162935} \end{array}$$

5. Divide

5. Divide $99\cdot450\dot{3}$ by $3\cdot6$ or $3\frac{6}{10} = 3\frac{3}{5} = \frac{18}{5}$.

$$\begin{array}{r} 11 \) \ 298\cdot36510 \\ \underline{27\cdot1241} \end{array}$$

6. Divide $4\cdot288215\dot{4}$ by $3\cdot5$ or $3\frac{5}{10} = \frac{7}{2}$.

$$\begin{array}{r} 32 \) \ 38\cdot5939363 \\ \underline{8 \ 9\cdot648} \\ 1\cdot206 \end{array}$$

7. Divide $4\cdot28215\dot{4}$ by $1\cdot20\dot{6}$.

$$\text{Here } 1\cdot20\dot{6} = 1\cdot2\frac{6}{10} = 1\cdot2\frac{3}{5} = \frac{298}{250}$$

$$\text{And } 4\cdot288215\dot{4} = 4\cdot2\frac{882154}{1000000} = \frac{42882154}{10000000}$$

$$\text{Then } \frac{42882154}{10000000} \times \frac{250}{298} = \frac{42882154}{307037198} = 3\cdot5$$

Or rather thus :

Having found $1\cdot20\dot{6} = \frac{298}{250} = \frac{149}{125}$, then

$$\begin{array}{r} 4\cdot2882154 \\ \underline{10} \end{array}$$

$$\begin{array}{r} 42\cdot882154 \\ \underline{3} \end{array}$$

$$\begin{array}{r} 128\cdot646464 \\ \underline{11} \end{array}$$

$$\begin{array}{r} 398 \) \ 1415\cdot11 \ (3\cdot5 \\ \underline{2211} \\ 221 \end{array}$$

IN-

INVOLUTION;

OR

RAISING OF POWERS.

A POWER is a number produced by multiplying any given number continually by itself a certain number of times.

Any number is called the first power of itself; if it be multiplied by itself, the product is called the second power, and sometimes the square; if this be multiplied by the first power again, the product is called the third power, and sometimes the cube; and if this be multiplied by the first power again, the product is called the fourth power; and so on: that is, the power is denominated from the number which exceeds the multiplications by 1.

Thus: 3 is the first power of 3.

$3 \times 3 = 9$ is the second power of 3.

$3 \times 3 \times 3 = 27$ is the third power of 3.

$3 \times 3 \times 3 \times 3 = 81$ is the fourth power of 3.

&c.

&c.

And in this manner may be calculated the following table.

TABLE of the First Twelve Powers of Numbers.

1st power	1	2	3	4	5	6	7	8	9
2d power	1	4	9	16	25	36	49	64	81
3d power	1	8	27	64	125	216	343	512	729
4th power	1	16	81	256	625	1296	2401	4096	6561
5th power	1	32	243	1024	3125	7776	16807	32768	59049
6th power	1	64	729	4096	15625	46656	117649	262144	531441
7th power	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th power	1	2048	177147	4194304	4828125	362797056	1977326743	859934592	31381059609
12th power	1	4096	531441	16777216	244140625	176782336	1384227201	68719476736	282420536481

The number which exceeds the multiplications by 1, is called the index or exponent of the power: so the index of the first power is 1, that of the second power is 2, that of the third is 3, and so on.

Powers are commonly denoted by writing their indices above the first power: so the second power of 3 is denoted thus, 3^2 ; the third power thus, 3^3 ; the fourth power thus, 3^4 ; and so on: also the 6th power of 503, thus, 503^6 .

Involution is the finding of powers; to do which, from their definition their evidently comes this rule.

R U L E.

Multiply the given number, or first power, continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

Note, 1. Because fractions are multiplied by taking the products of their numerators and of their denominators, they will be involved by raising each of their terms to the power required. And if a mixt number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

2. The raising of powers may be sometimes shortened by working according to this observation, viz. whatever two or more powers are multiplied together, their product is the power whose index is the sum of the indices of the factors; or if a power be multiplied by itself, the product will be the power whose index is double of that which is multiplied; so if I would find the sixth power, I might multiply the given number twice by itself for the third power, then the third power into itself would give the sixth power; or if I would find the seventh power; I might first find the third and fourth, and their product would be the seventh; or lastly, if I would find the eighth

C

power,

power, I might first find the second, then the second into itself would be the fourth, and this into itself would be the eighth.

EXAMPLE 1.

For the square of 45.

$$\begin{array}{r}
 45 \text{ 1st power.} \\
 45 \\
 \hline
 225 \\
 180 \\
 \hline
 2025 = 45^2
 \end{array}$$

EXAMPLE 2.

For the square of .027.

$$\begin{array}{r}
 .027 \\
 .027 \\
 \hline
 189 \\
 54 \\
 \hline
 .000729 = .027^2
 \end{array}$$

EXAMPLE 3.

For the cube of 3.5.

$$\begin{array}{r}
 3.5 \\
 3.5 \\
 \hline
 175 \\
 105 \\
 \hline
 12.25 \\
 3.5 \\
 \hline
 6125 \\
 3675 \\
 \hline
 42.875 = 3.5^3
 \end{array}$$

EXAMPLE 4.

For the fourth power of 5.1.

$$\begin{array}{r}
 5.1 \\
 5.1 \\
 \hline
 5.1 \\
 255 \\
 \hline
 26.01 = 5.1^2 \\
 26.01 \text{ ditto} \\
 \hline
 2601 \\
 15606 \\
 5202 \\
 \hline
 676.5201 = 5.1^4
 \end{array}$$

EXAMPLE 5.

For the fifth power of $\cdot 29$.

$$\begin{array}{r}
 \cdot 29 \\
 \cdot 29 \\
 \hline
 261 \\
 58 \\
 \hline
 \cdot 0841 = 29^2 \\
 \cdot 0841 \text{ ditto} \\
 \hline
 841 \\
 3364 \\
 6728 \\
 \hline
 \cdot 0707281 = \cdot 29^4 \\
 \cdot 29 = 1st \\
 \hline
 6365529 \\
 1414562 \\
 \hline
 \cdot 0020511149 = \cdot 29^5
 \end{array}$$

EXAMPLE 6.

For the sixth power of $2\cdot 6$.

$$\begin{array}{r}
 2\cdot 6 \\
 2\cdot 6 \\
 \hline
 156 \\
 52 \\
 \hline
 676 = 2\cdot 6^2 \\
 2\cdot 6 \\
 \hline
 4056 \\
 1352 \\
 \hline
 17\cdot 576 = 2\cdot 6^3 \\
 17\cdot 576 \text{ ditto} \\
 \hline
 105456 \\
 123032 \\
 87880 \\
 123032 \\
 17576 \\
 \hline
 308\cdot 915776 = 2\cdot 6^6
 \end{array}$$

Ex. 7. The square of $\frac{2}{3}$ is $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.Ex. 8. The cube of $\frac{2}{3}$ is $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$.Ex. 9. The square of $3\frac{2}{3}$ or $\frac{17}{3}$ is $\frac{17}{3} \times \frac{17}{3} = \frac{289}{9}$
 $= 11\frac{1}{3} = 11\cdot 56$.

EVOLUTION;

OR

EXTRACTION OF ROOTS.

The root of any given number, or power, is such a number, as being multiplied by itself a certain number of times, will produce the power; and it is denominated the first, second, third, fourth, &c. root, respectively, as the number of multiplications made of it to produce the given power, is 0, 1, 2, 3, &c; that is, the name of the root is taken from the number which exceeds the multiplications by 1, like the name of the power in involution.

The index of the root, like that of the power in involution, is 1 more than the number of the multiplications necessary to produce the power or given number. So 2 is the index of the second or square root; and 3 the index of the 3d or cubic root; and 4 the index of the 4th root; and so on.

Roots are sometimes denoted by writing $\sqrt{}$ before the power, with the index of the root against it: so the third root of 50 is $\sqrt[3]{50}$, and the second root of it is $\sqrt{\sqrt{50}}$, the index 2 being omitted; which index is always understood when a root is named or written without one. But if the power be expressed by several numbers with the sign + or —, &c. between them, then a line is drawn from the top of the sign of the root, or radical sign, over all the parts of it; so the third root of $47 - 15$, is $\sqrt[3]{47 - 15}$. And sometimes roots are designed like powers, with the reciprocal of the index of the root above

above the given number. So the root of 3 is $3^{\frac{1}{2}}$, the root of 50 is $50^{\frac{1}{2}}$, and the third root of it is $50^{\frac{1}{3}}$; also the third root of $47 - 15$ is $\sqrt[3]{47 - 15}$. And this method of notation has justly prevailed in the modern algebra; because such roots, being considered as fractional powers, need no other directions for any operations to be made with them, but those for integral powers.

A number is called a complete power of any kind, when its root of the same kind can be accurately extracted; but if not, the number is called an imperfect power, and its root a surd or irrational quantity. So 4 is a complete power of the second kind, its root being 2; but an imperfect power of the third kind, its third root being a surd quantity, which cannot be accurately extracted.

Evolution is the finding of the roots of numbers, either accurately, or in decimals to any proposed degree of accuracy.

The power is first to be prepared for extraction, or evolution, by dividing it, by means of points or commas, from the place of units, to the left hand in integers, and to the right in decimal fractions, into periods, containing each as many places of figures as are denoted by the index of the root, if the power contain a complete number of such periods; that is, each period to have two figures for the square root, three for the cube root, four for the fourth root, and so on. And when the last period in decimals is not complete, ciphers are added to complete it.

Note, The root will contain just as many places of figures, as there are periods or points in the given power; and they will be integers or decimals, respectively, as the periods are so, from which they are found, or to which they correspond; that is, there will be as many integer or decimal figures in the root, as there are periods of integers or decimals in the given number.

TO EXTRACT THE SQUARE ROOT.

1. Having divided the given number into periods of two figures each, find, from the table of powers in page 24, or otherwise, a square number either equal to, or the next less than the first period, which subtract from it, and place the root of the square on the right of the given number, after the manner of a quotient in division, for the first figure of the root required.
 2. To the remainder annex the second period for a dividend; and on the left thereof set the double of the root already found, after the manner of a divisor.
 3. Find how often the divisor is contained in the dividend, wanting its last figure on the right hand; place that number for the next figure in the quotient, and on the right of the divisor, as also below the same.
 4. Multiply the whole increased divisor by it, placing the product below the dividend, and subtract it from it, and to the remainder bring down the next period, for a new dividend; to which, as before, find a divisor by doubling the figures already found in the root; and from these find the next figure of the root, as in the last article; continuing the operation still in the same manner till all the periods be used, or as far as you please.
- Note.* Instead of doubling the root, to find the new divisors, you may add the last divisor to the figure below it.

To prove the work, multiply the root by itself, and to the product add the remainder, and the sum will be the given number.

Ex.

Ex. 1. To extract the root of 17·3056.

Having divided the given number into three periods, namely 17, and 30, and 56, I find that 16 is the next square to 17, the first period, which set below, and subtracting, 1 remains, to which bring down 30, the next period, and it makes 130 for a dividend. Then 4, the root of 16, is set on the right of the given number for the first figure of the root, and its double, or 8, on the left of the dividend for the first figure of the divisor; which being once contained in 13, the dividend wanting its last figure, gives 1 for the next figure of the root, which 1 is accordingly set in the root, making 4·1, and in the divisor making 81, as also below the same. These multiplied make also 81, which set below the dividend, and subtracting, we have 49 remaining, to which the last period 56 being brought down, it makes 4956 for the new dividend. Then, for a new divisor, either double the root 4·1, or else, which is easier, to the last divisor add the figure 1 standing below it, and either way gives 82 for the first part of the new divisor. This 82 is 6 times contained in 495, and therefore 6 is the next figure, to set in the root, and in the divisor, as also below the same; which being then multiplied by it, gives 4956, the same as the dividend; therefore nothing remains, and 4·16 is the root of 17·3056, as required.

$$\begin{array}{r}
 17\cdot30,56 \quad (4\cdot16 \\
 \underline{16} \\
 81 \mid 130 \\
 \underline{1} \mid 81 \\
 825 \mid 4956 \\
 \underline{6} \mid 4956
 \end{array}$$

EXAMPLE 2.

For the Root of 2025.

20,25 (45 root
16

$$\begin{array}{r|l} 85 & 425 \\ 5 & 425 \end{array}$$

EXAMPLE 3.

For the Root of .000729.

.00,07,29 (.027 root
4

$$\begin{array}{r|l} 47 & 329 \\ 7 & 329 \end{array}$$

Note, When all the periods of the given number are brought down and used, and more figures are required to be found, the operation may be continued by adding as many periods of ciphers as we please, namely, annexing always two ciphers at once to each dividend. And when the root is to be extracted to a great number of places, the work may be much abbreviated thus: having proceeded in the extraction after the common method till you have found one more than half the required number of figures in the root, the rest may be found by dividing the last remainder by its corresponding divisor, annexing a cipher to every dividend, as in division of decimals; or rather, without annexing ciphers, by omitting continually the right-hand figure of the divisor, after the manner of the third contraction in division of decimals in page 10.

So the operation for the root of 2, to 12 or 13 places, may be thus.

EXAMPLE 4.

2 (1'414213562373 root
1

$$\begin{array}{r|l} 24 & 100 \\ 4 & 96 \end{array}$$

$$\begin{array}{r|l} 281 & 400 \\ 1 & 281 \end{array}$$

$$\begin{array}{r|l} 2824 & 11900 \\ 4 & 11296 \end{array}$$

$$\begin{array}{r|l} 28282 & 60400 \\ 2 & 56564 \end{array}$$

$$\begin{array}{r|l} 282841 & 383600 \\ 1 & 282841 \end{array}$$

$$\begin{array}{r|l} 2828423 & 10075900 \\ 3 & 8485269 \end{array}$$

$$\begin{array}{r} 2828426) 1590631 (562373 \\ \dots\dots 176418 \\ 6712 \\ 1055 \\ 206 \\ 8 \end{array}$$

Here, having found the first seven figures 1'4'4213 by the common extraction, by adding always periods of ciphers, the last six figures 562373 are found by the method of contracted division in decimals, without adding ciphers to the remainder, but only pointing off a figure at each time from the last divisor.

And the same for the two following examples.

EXAMPLE 5.

For the root of 3

3 (1.732051 root
1

$$\begin{array}{r|l} 27 & 200 \\ 7 & 189 \end{array}$$

$$\begin{array}{r|l} 343 & 1100 \\ 3 & 1029 \end{array}$$

$$\begin{array}{r|l} 3462 & 7100 \\ 2 & 6924 \end{array}$$

$$\begin{array}{r} 3464 \end{array}) \begin{array}{r} 176 \\ \dots \end{array} (\begin{array}{r} 051 \\ 3 \end{array}$$

EXAMPLE 6.

For the root of 5.

5 (2.236068 root
4

$$\begin{array}{r|l} 42 & 100 \\ 2 & 84 \end{array}$$

$$\begin{array}{r|l} 443 & 1600 \\ 3 & 1329 \end{array}$$

$$\begin{array}{r|l} 4466 & 27100 \\ 6 & 26796 \end{array}$$

$$\begin{array}{r} 4472 \end{array}) \begin{array}{r} 304 \\ \dots \end{array} (\begin{array}{r} 068 \\ 36 \end{array}$$

In like manner may be found the following Roots.

The root of 6 is 2.449490

The root of 7 is 2.645751

The root of 10 is 3.162278

The root of 11 is 3.316625

RULES for the Square Roots of Vulgar Fractions and Mixt Numbers.

First prepare all vulgar fractions, by reducing them to their least terms, both for this and all other roots. Then,

1. Take the root of the numerator and of the denominator, for the respective terms of the root required. And this is the best way if the denominator be a complete power. But if it be not,

2. Multiply the numerator and denominator together; take the root of the product; this root being made

made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional root required.

$$\text{That is, } \sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b} = \frac{b}{\sqrt{ab}}$$

And this rule will serve whether the root be finite or infinite. Or,

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixt numbers may be either reduced to improper fractions, and extracted by the first or second rule; or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

Ex. 1. $\sqrt{\frac{2}{3}}$ is $\frac{2}{3}$

Ex. 2. $\sqrt{\frac{27}{4}}$ or $\sqrt{\frac{9}{4}}$ is $\frac{3}{2}$

Ex. 3. For the root of $\frac{9}{16}$

Here $\frac{9}{16}$ or $\frac{3}{4}$ is .75 (.866025 root
64

166	1100
6	996
1726	10400
6	10356
1732)	44 (025
...	9

Ex. 4. For the root of $\frac{3}{16}$

c 6

Here

Here $\frac{1}{12}$ is = $\cdot 4166$ ($\cdot 645497$ root

$$\begin{array}{r}
 36 \\
 \hline
 124 \overline{) 566} \\
 \underline{4 496} \\
 1285 \overline{) 7066} \\
 \underline{5 6425} \\
 1290 \overline{) 641 497} \\
 \underline{ 125} \\
 9
 \end{array}$$

TO FIND A MEAN PROPORTIONAL.

There are various uses of the square root, one of which is to find a mean proportional between any two numbers, which is performed thus: Multiply the two given numbers together, then extract the square root out of their product, and it will be the mean proportional sought.

Ex. 1. To find a Mean Proportional between 3 and 12.

Here $3 \times 12 = 36$.

And $\sqrt{36}$ is 6, the mean proportional sought.

For $3 : 6 :: 6 : 12$.

Ex. 2. To find a Mean between 2 and 5.

Here $2 \times 5 = 10$ ($3\cdot 162278$ the mean required.

$$\begin{array}{r}
 9 \\
 \hline
 61 \overline{) 100} \\
 \underline{1 61} \\
 626 \overline{) 3900} \\
 \underline{6 3756} \\
 6322 \overline{) 14400} \\
 \underline{2 12644} \\
 6324 \overline{) 1756 278} \\
 \underline{ 491} \\
 49
 \end{array}$$

Note.

Note. By means of the square root also we readily find the 4th root, or the 8th root, or the 16th root, &c, that is, the root of any power whose index is some power of the number 2: namely, by extracting so often the square root as is denoted by the index of that power of 2; that is, two extractions for the 4th root, three for the 8th root, and so on.

Thus for the 4th root of $97^{\circ}41$.

$ \begin{array}{r} 97^{\circ}41,00,00 \\ 81 \\ \hline 188 \overline{) 1641} \\ 8 \overline{) 1504} \\ \hline 1966 \overline{) 13700} \\ 6 \overline{) 11796} \\ \hline 19729 \overline{) 190400} \\ 9 \overline{) 177561} \\ \hline 19738 \overline{) 12839} \\ 996 \\ 9 \end{array} $	$ \begin{array}{r} (9^{\circ}86,96,50,50 \text{ (3}^{\circ}14159999 \text{ anf.} \\ 9 \\ \hline 61 \overline{) 86} \\ 1 \overline{) 61} \\ \hline 624 \overline{) 2596} \\ 4 \overline{) 2496} \\ \hline 6281 \overline{) 10050} \\ 1 \overline{) 6281} \\ \hline 62825 \overline{) 376950} \\ 5 \overline{) 314125} \\ \hline 62830 \overline{) 62825} \text{ (9999} \\ 6278 \\ 623 \\ 58 \end{array} $
--	---

So that the 4th root of $97^{\circ}41$ is $3^{\circ}14159999$, which expresses the circumference of a circle whose diameter is 1 nearly.

OTHER EXAMPLES.

The 4th root of $21035^{\circ}8$ is $12^{\circ}0431407$.

The 4th root of - 2 is $1^{\circ}189207$.

TO

TO EXTRACT THE CUBE ROOT.

RULE I.

1. Point the given number into periods of three places each, beginning at units ; and there will be as many integral places in the root, as there are points over the integers in the given number.

2. Seek the greatest cube in the left-hand period ; write the root in the quotient, and the cube under the period ; from which subtract it, and to the remainder bring down the next period : Call this the resolvend, under which draw a line.

3. Under the resolvend, write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former ; under the triple square of the root, write the triple root, removed one place to the right ; and the sum of these two lines call a divisor ; under which draw a line.

4. Seek how often this divisor may be had in the resolvend, its right-hand place excepted, and write the result in the quotient.

5. Under the divisor, write the product of the triple square of the root by the last quotient figure, setting the units place of this line, under that of tens in the divisor ; under this line, write the product of the triple root by the square of the last quotient figure, let this line be removed one place beyond the right of the former ; and under this line, removed one place forward to the right, set the cube of the last quotient figure ; the sum of these three lines call the subtrahend, under which draw a line.

6. Subtract the subtrahend from the resolvend ; to the remainder bring down the next period for a new resolvend ; the divisor to this, must be the triple square of all the quotient added to the triple thereof, &c, as in the third article, &c.

EXAMPLE I.

What is the cube root of 48228544?

$$\begin{array}{r}
 48228544 \quad (354 \\
 \underline{27} \\
 21228 \quad \text{Resolvend} \\
 \hline
 \text{add } \left\{ \begin{array}{l} 27 \\ 09 \end{array} \right. \quad \left. \begin{array}{l} \text{Triple square of 3} \\ \text{Triple of 3.} \end{array} \right\} \text{ the root.} \\
 \hline
 279 \quad \text{Divisor} \\
 \hline
 \text{add } \left\{ \begin{array}{l} 162 \\ 324 \\ 216 \end{array} \right. \quad \left. \begin{array}{l} \text{Triple square of 3 multiplied by 6} \\ \text{Triple of 3 multiplied by square of 6} \\ \text{Cube of 6} \end{array} \right\} \\
 \hline
 19656 \quad \text{Subtrahend} \\
 \hline
 1572544 \quad \text{Resolvend} \\
 \hline
 \text{add } \left\{ \begin{array}{l} 3888 \\ 108 \end{array} \right. \quad \left. \begin{array}{l} \text{Triple square of 36} \\ \text{Triple of 36} \end{array} \right\} \text{ the root.} \\
 \hline
 38988 \quad \text{Divisor} \\
 \hline
 \text{add } \left\{ \begin{array}{l} 15552 \\ 1728 \\ 64 \end{array} \right. \quad \left. \begin{array}{l} \text{Triple square of 36 mult. by 4} \\ \text{Triple of 36 mult. by square of 4} \\ \text{Cube of 4} \end{array} \right\} \\
 \hline
 1572544 \quad \text{Subtrahend}
 \end{array}$$

If the work of this example be well considered, and compared with the foregoing rule, it will be easy to conceive how any other example of the same kind may be wrought. And here observe, that when the cube root is extracted to more than two places, there is a necessity of doing some work upon a spare piece of paper, in order to

to come at the root's triple square, and the product of the triple root by the square of the quotient figure, &c.

In this example, the given number is a cubic number, and therefore at the end of the operation there remained nothing: for 364 multiplied by 364, and the product multiplied by 364 again, gives 48228544, the given number.

But if the number given be not a cubic number; then, to the last remainder always bring down three ciphers, and work anew for a decimal fraction if needful.

MORE EXAMPLES.

What is the cube root of

$$\left. \begin{array}{r} 389017 \\ 1092727 \\ 2705403608 \\ 219365327791 \\ 122615327232 \end{array} \right\} \text{Answers.} \quad \left\{ \begin{array}{l} 73 \\ 103 \\ 3002 \\ 6031 \\ 4968 \end{array} \right.$$

These examples are all performed in the same manner as the foregoing one.

TO FIND TWO MEAN PROPORTIONALS.

There are many uses of the cube root; one is to find two mean proportionals between two given numbers: which is performed thus:

Divide the greater extreme by the less, and the cube root of the quotient multiplied by the less extreme, gives the less mean. Multiply the said cube root by the less mean, and the product is the greater mean proportional.

Note, This is only understood of those numbers that are in continued geometric proportion.

EXAMPLE I.

What are the two mean proportionals between 4 and 108?

108

108 divided by 4 gives 27, whose cube root is 3: and the less extreme 4, multiplied by 3, gives 12 for the less mean; and 12 multiplied by the said root 3, gives 36 for the greater mean.

For 4 is to 12 as 36 to 108.

EXAMPLE II.

To find two geometrical means between 8 and 1728?

Now 8) 1728 (216, whose cube root is 6. And 6 times 8 is 48, the less mean, and 6 times 48 is 288, the greater mean.

For 8 is to 48 as 288 to 1728.

If the rule already given for the cube root be thought too tedious, the following one will be found much more easy and ready for use.

RULE II.

FOR THE CUBE ROOT.

1. By trials take the nearest rational cube to the given cube or number, and call it the assumed cube.

2. Then the sum of the given number and double the assumed cube, will be to the sum of the assumed cube, and double the given number, as the root of the assumed cube, is to the root required, nearly. Or as the first sum is to the difference of the given and assumed cube, so is the assumed root, to the difference of the roots nearly.

3. Again, by using, in like manner, the cube of the root last found as a new assumed cube, another root will be obtained still nearer. And so on as far as we please; using always the cube of the last-found root, for the assumed cube. -

EXAM-

EXAMPLE.

To find the cube root of 21035·8.

Here we soon find that the root lies between 20 and 30, and then between 27 and 28. Taking therefore 27, its cube is 19683 the assumed cube. Then

$$\begin{array}{r} 19683 \\ 2 \\ \hline 39366 \\ 21035 \cdot 8 \\ \hline \end{array} \quad \begin{array}{r} 21035 \cdot 8 \\ 2 \\ \hline 42071 \cdot 6 \\ 19683 \\ \hline \end{array}$$

$$\text{As } 60401 \cdot 8 : 61754 \cdot 6 :: 27 : 27 \cdot 6047$$

$$\begin{array}{r} 27 \\ \hline 4322822 \\ 1235092 \\ \hline \end{array}$$

60401·8) 1667374·2 (27·6047 the root nearly.

$$\begin{array}{r} 459338 \\ 36525 \\ 284 \\ 42 \end{array}$$

Again, for a second operation, the cube of this root is 21035·318645155823, and the process by the latter method will be thus:

$$\begin{array}{r} 21035 \cdot 318645 \text{ \&c.} \\ 2 \end{array}$$

$$\begin{array}{r} 42070 \cdot 637290 \\ 21035 \cdot 8 \\ \hline \end{array} \quad \begin{array}{r} 21035 \cdot 8 \\ 21035 \cdot 318645 \text{ \&c.} \\ \hline \end{array}$$

$$\text{As } 63106 \cdot 437290 : \text{dif. } 481355 :: 27 \cdot 6047 : \text{the dif. } 000210834$$

conseq. the root req. is 27·604910834

TO

TO EXTRACT ANY ROOT WHATEVER.

Let G be the given power or number, n the index of the power, A the assumed power, r its root, R the required root of G . Then

As the sum of $n + 1$ times A and $n - 1$ times G , is to the sum of $n + 1$ times G and $n - 1$ times A , so is the assumed root r , to the required root R .

Or, as half the said sum of $n + 1$ times A and $n - 1$ times G , is to the difference between the given and assumed powers, so is the assumed root r , to the difference between the true and assumed roots: which difference added or subtracted, gives the true root nearly.

That is, $\overline{n + 1. A + n - 1. G} : \overline{n + 1. G + n - 1. A} :: r : R$.

Or, $\overline{n + 1. \frac{1}{2} A + n - 1. \frac{1}{2} G} : A \oslash G :: r : R \oslash r$.

And the operation may be repeated as often as we please, by using always the last found root for the assumed root, and its n th power for the assumed power A .

EXAMPLE.

To extract the 5th Root of 21035.8.

Here it appears that the 5th root is between 7.3 and 7.4. Taking 7.3, its 5th power is 20730.71593. Hence then we have

$G =$

$$G = 21035^{\circ}8; r = 7^{\circ}3; n = 5; \frac{1}{2} \cdot \frac{n+1}{n-1} = 3;$$

$$A = 20730^{\circ}716 \quad \text{and } \frac{1}{2} \cdot \frac{n+1}{n-1} = 2$$

$$G - A = 305^{\circ}084$$

$$A = 20730^{\circ}716$$

3

$$3A = 62192^{\circ}148$$

$$2G = 42071^{\circ}6$$

$$G = 21035^{\circ}8$$

2

$$42071^{\circ}6$$

$$As \ 104263^{\circ}7 : 305^{\circ}084 :: 7^{\circ}3 : ^{\circ}0213605$$

7.3

$$915252$$

$$2135588$$

$$104263^{\circ}7) 2227^{\circ}1132 \quad (^{\circ}0213605$$

$$14184$$

$$3758$$

$$630$$

5

$$7.3 = r \text{ add}$$

$7.321360 = r$ the
root true to the
last figure.

OTHER EXAMPLES.

1. What is the 3d root of 2? Ans. $1^{\circ}259921$.
2. What is the 4th root of 2? Ans. $1^{\circ}189207$.
3. What is the 4th root of $97^{\circ}41$? Ans. $3^{\circ}141599$.
4. What is the 5th root of 2? Ans. $1^{\circ}148699$.
5. What is the 6th root of $21035^{\circ}8$? Ans. $5^{\circ}254037$.
6. What is the 6th root of 2? Ans. $1^{\circ}122462$.
7. What is the 7th root of $21035^{\circ}8$? Ans. $4^{\circ}145392$.
8. What is the 7th root of 2? Ans. $1^{\circ}104089$.
9. What is the 8th root of $21035^{\circ}8$? Ans. $3^{\circ}470323$.
10. What is the 8th root of 2? Ans. $1^{\circ}090508$.
11. What is the 9th root of $21035^{\circ}8$? Ans. $3^{\circ}022239$.
12. What is the 9th root of 2? Ans. $1^{\circ}080059$.

GENERAL RULES for extracting any Root out of a Vulgar Fraction or Mixt Number.

1. If the given fraction have a finite root of the kind required, it is best to extract the root out of the numerator and denominator, for the terms of the root required.

2. But if the fraction be not a complete power, it may be thrown into a decimal, and then extracted. Or,

3. Take either of the terms of the given fraction for the corresponding term of the root; and for the other term of the root, extract the required root of the product, arising from the multiplication of such a power of the first assigned term of the root whose index is less by 1 than that of the given power, by the other term of the given number.

This rule will do when the root is either finite or infinite.

$$\text{That is, } \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{ab^{n-1}}}{b} = \frac{a}{\sqrt[n]{ba^{n-1}}}.$$

4. Mixt numbers may be reduced either to improper fractions or decimals, and then extracted.

EXAMPLES.

1. What is the cube root of $\frac{8}{27}$? Anf. - - - $\frac{2}{3}$.
2. What is the fourth root of $\frac{80}{403}$? Anf. - - - $\frac{2}{3}$.
3. What is the cube root of $\frac{1}{2}$? Anf. $\cdot 7937005$.
4. What is the cube root of $2\frac{10}{27}$? Anf. $\frac{4}{3}$ or $1\frac{1}{3}$.
5. What is the third root of $7\frac{1}{3}$? Anf. $1\cdot 930979$.

DUO.

DUODECIMALS;

OR

CROSS MULTIPLICATION.

DUODECIMALS are the calculation by feet, inches, and parts, and are so called, because they decrease by twelves, from the place of feet, towards the right hand. Inches are sometimes called primes, and are marked thus'; the next division after inches are called parts, or seconds, and are marked thus"; the next are thirds, and marked thus'''; and so on.

This rule is otherwise called Cross Multiplication, because the factors are sometimes multiplied cross ways. And it is commonly used by workmen and artificers in casting up the contents of their work; the dimensions being taken in feet, inches, and parts; though a much better way would be by a decimal scale of divisions.

RULE I.

1. Under the multiplicand write the same names or denominations of the multiplier; that is, feet under feet, inches under inches, parts under parts, &c.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and set each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.

3. In the same manner multiply every term in the multiplicand by the inches in the multiplier, and set the result of each term one place removed to the right of those in the multiplicand.

1

4. Pro-

4. Proceed in like manner with the seconds, and all the rest of the denominations, if there be any more, setting the product of each line always one place more towards the right hand than the line next before, and the sum of all the lines will be the whole product required.

Or the denominations of the particular products will be as follow :

Feet by feet, give feet.
 Feet by primes, give primes.
 Feet by seconds, give seconds,
 &c.

Primes by primes, give seconds.
 Primes by seconds, give thirds.
 Primes by thirds, give fourths,
 &c.

Seconds by seconds, give fourths.
 Seconds by thirds, give fifths.
 Seconds by fourths, give sixths,
 &c.

Thirds by thirds, give sixths.
 Thirds by fourths, give sevenths.
 Thirds by fifths, give eighths,
 &c.

In general thus:

When feet are concerned, the product is of the same denomination with the term multiplying the feet.

When feet are not concerned, the name of the product is expressed by the sum of the indices of the two factors.

Ex. 1

Ex. 1. Multiply

f	'	"		f	'	"
10	4	5	by	7	8	6
7	8	7				
<hr/>						
72	6	11				
6	10	11	4			
	5	2	2	6		
<hr/>						
79	11	0	6	6		Answer
<hr/>						

RULE II.

When the feet in the multiplicand are expressed by a large number.

Multiply first by the feet of the multiplier, as before.

Then, instead of multiplying by the inches and parts, &c. proceed as in the Rule of Practice, by taking such aliquot parts of the multiplicand as correspond with the inches and seconds, &c. of the multiplier. Then the sum of them all will be the product required.

Ex. 2. Multiply

f	'	"		f	'	"
240	10	8	by	9	4	6
9	4	6				
<hr/>						
2168	0	0				
4 = $\frac{1}{3}$	-	-	80	3	6	8
6 = $\frac{1}{8}$	-	-	10	0	5	4
<hr/>						
2258	4	0	0			Answer.
<hr/>						

RULE III.

If the feet in both the multiplicand and multiplier be large numbers.

Multiply the feet only into each other: then, for the inches and seconds in the multiplier, take parts of the multi-

multiplicand; and for the inches and secouds of the multiplicand, take aliquot parts of the feet only in the multiplier. Then the sum of all will be the whole product.

Ex. 3. Multiply $\overset{f}{3} \overset{i}{6} \overset{''}{8} \overset{''' }{5}$ by $\overset{f}{1} \overset{i}{3} \overset{''}{7} \overset{''' }{8} \overset{'''' }{4}$

$$\begin{array}{r}
 2576 \\
 1104. \\
 368.. \\
 \hline
 6' = \frac{1}{2} - - 184 \quad 3 \quad 8 \quad 6 \\
 2' = \frac{1}{3} - - 61 \quad 5 \quad 2 \quad 10 \\
 4'' = \frac{1}{3} - - 10 \quad 2 \quad 10 \quad 5 \quad 8 \\
 6' = \frac{1}{2} - - 68 \quad 6 \quad . \quad . \quad . \\
 1' = \frac{1}{3} - - 11 \quad 5 \quad . \quad . \quad . \\
 4'' = \frac{1}{3} - - 3 \quad 9 \quad 8 \quad . \quad . \\
 1'' = \frac{1}{4} - - 11 \quad 5 \quad . \quad . \quad . \\
 \hline
 50756 \quad 7 \quad 10 \quad 9 \quad 8 \quad \text{Answer}
 \end{array}$$

OTHER EXAMPLES.

	Questions.					Answers.						
	f	i	''	'''	iv	f	i	''	'''	iv	v	vi
4. Mult.	4	7	.	.	.	29	0	4
by	6	4	.	.	.							
5. Mult.	14	9	9	.	.	66	4	6
by	4	6	.	.	.							
6. Mult.	4	7	8	.	.	44	0	10
by	9	6	.	.	.							
7. Mult.	7	8	6	.	.	79	11	0	6	6	.	.
by	10	4	5	.	.							
8. Mult.	39	10	7	.	.	745	6	10	2	4	.	.
by	18	8	4	.	.							
9. Mult.	44	2	9	2	4	126	2	10	8	10	11	.
by	2	10	3	.	.							
10. Mult.	24	10	8	7	5	233	4	5	9	6	4	.
by	9	4	6	.	.							

D

A TABL

T A B L E

OF

SQUARES AND CUBES, ALSO SQUARE ROOTS AND
CUBE ROOTS.

Num- ber.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1'0000000	1'000000
2	4	8	1'4142136	1'259921
3	9	27	1'7320508	1'442250
4	16	64	2'0000000	1'587401
5	25	125	2'2360680	1'709976
6	36	216	2'4494897	1'817121
7	49	343	2'6457513	1'912933
8	64	512	2'8284271	2'000000
9	81	729	3'0000000	2'080084
10	100	1000	3'1622777	2'154435
11	121	1331	3'3166248	2'223980
12	144	1728	3'4641016	2'289428
13	169	2197	3'6055513	2'351335
14	196	2744	3'7416574	2'410142
15	225	3375	3'8729833	2'466212
16	256	4096	4'0000000	2'519842
17	289	4913	4'1231056	2'571282
18	324	5832	4'2426407	2'620741
19	361	6859	4'3588989	2'668402
20	400	8000	4'4721360	2'714416
21	441	9261	4'5825757	2'758923
22	484	10648	4'6904158	2'802039
23	529	12167	4'7958315	2'843867
24	576	13824	4'8939795	2'884469
25	625	15625	5'0000000	2'924018

A TABLE OF SQUARES, &c. 51

Num- ber.	Square.	Cube.	Square Root.	Cube Rcot.
26	676	17576	5.0990195	2.962496
27	729	19683	5.1961524	3.000000
28	784	21952	5.2915026	3.036589
29	841	24389	5.3851648	3.072317
30	900	27000	5.4772256	3.107732
31	961	29791	5.5677644	3.141381
32	1024	32768	5.6568542	3.174802
33	1089	35937	5.7445626	3.207534
34	1156	39304	5.8309519	3.23612
35	1225	42875	5.9160798	3.271066
36	1296	46656	6.0000000	3.311927
37	1369	50653	6.0827625	3.332222
38	1444	54872	6.1644140	3.361975
39	1521	59319	6.2449980	3.391211
40	1600	64000	6.3245553	3.419952
41	1681	68921	6.401242	3.448217
42	1764	74088	6.4807407	3.476027
43	1849	79507	6.5574385	3.501398
44	1936	85184	6.6332496	3.530348
45	2025	91125	6.7082039	3.556893
46	2116	97336	6.7823300	3.583048
47	2209	103823	6.8556546	3.608826
48	2304	110592	6.9282032	3.634241
49	2401	117649	7.0000000	3.659306
50	2500	125000	7.0710678	3.684031
51	2601	132651	7.1414284	3.708430
52	2704	140608	7.2111026	3.732511
53	2809	148877	7.2801099	3.756286
54	2916	157464	7.3484692	3.779763
55	3025	166375	7.4161985	3.802953
56	3136	175616	7.483148	3.82562
57	3249	185193	7.5498344	3.848501
58	3364	195112	7.6157731	3.870877
59	3481	205379	7.6811457	3.892996
60	3600	216000	7.7459667	3.914867

Num- ber.	Square.	Cube.	Square Root.	Cube Root.
61	3721	226981	7·8102497	3·936497
62	3844	238328	7·8740079	3·957892
63	3969	250047	7·9372539	3·979057
64	4096	262144	8·0000000	4·000000
65	4225	274625	8·0622577	4·020726
66	4356	287496	8·1240384	4·041240
67	4489	300763	8·1853528	4·061548
68	4624	314432	8·2462113	4·081656
69	4761	328509	8·3066239	4·101566
70	4900	343000	8·3666003	4·121285
71	5041	357911	8·4261498	4·140818
72	5184	373248	8·4852814	4·160168
73	5329	389017	8·5440037	4·179339
74	5476	405224	8·6023253	4·198336
75	5625	421875	8·6602540	4·217163
76	5776	438976	8·7177979	4·235824
77	5929	456533	8·7749644	4·254321
78	6084	474552	8·8317609	4·272059
79	6241	493039	8·8881944	4·290841
80	6400	512000	8·9442719	4·308870
81	6561	531441	9·0000000	4·326749
82	6724	551368	9·0553851	4·344481
83	6889	571787	9·1104336	4·362071
84	7056	592704	9·1651514	4·379519
85	7225	614125	9·2195445	4·396830
86	7396	636056	9·2736185	4·414005
87	7569	658503	9·3273791	4·431047
88	7744	681472	9·3828315	4·447960
89	7921	704969	9·4339811	4·464745
90	8100	729000	9·4868330	4·481405
91	8281	753571	9·5393920	4·497942
92	8464	778688	9·5916630	4·514357
93	8649	804357	9·6436508	4·530655
94	8836	830584	9·6953597	4·546836
95	9025	857375	9·7467943	4·562903

Num- ber.	Square.	Cube.	Square Root.	Cube Root.
96	9216	884736	9'7979590	4'578857
97	9409	912673	9'8488578	4'594701
98	9604	941192	9'8994949	4'610436
99	9801	970299	9'9498744	4'626265
100	10000	1000000	10'0000000	4'641589
101	10201	1030301	10'0498756	4'657010
102	10404	1061208	10'0995049	4'672330
103	10609	1092727	10'1488916	4'687548
104	10816	1124864	10'1980390	4'702669
105	11025	1157625	10'2469508	4'717094
106	11236	1191016	10'2956301	4'732624
107	11449	1225043	10'3440804	4'747459
108	11664	1259712	10'3923048	4'762203
109	11881	1295029	10'4403065	4'776856
110	12100	1331000	10'4880885	4'791420
111	12321	1367631	10'5356538	4'805896
112	12544	1404928	10'5830052	4'820284
113	12769	1442897	10'6301458	4'834588
114	12996	1481544	10'6770783	4'848808
115	13225	1520875	10'7238053	4'862944
116	13456	1560896	10'7703296	4'876999
117	13689	1601613	10'8166538	4'890973
118	13924	1643032	10'8627805	4'904868
119	14161	1685159	10'9087121	4'918685
120	14400	1728000	10'9544512	4'932424
121	14641	1771561	11'0000000	4'946088
122	14884	1815848	11'0453610	4'959675
123	15129	1860867	11'0905365	4'973190
124	15376	1906624	11'1355287	4'986631
125	15625	1953125	11'1803399	5'000000
126	15876	2000376	11'2249722	5'013298
127	16129	2048383	11'2694277	5'026526
128	16384	2097152	11'3137085	5'039684
129	16641	2146689	11'3578167	5'052774
130	16900	2197000	11'4017543	5'065797

Num- ber.	Square.	Cube.	Square Root.	Cube Root.
131	17161	2248091	11'4455231	5'078753
132	17424	2299968	11'4891253	5'091643
133	17689	2352637	11'5325626	5'104469
134	17956	2406104	11'5758369	5'117230
135	18225	2460375	11'6189500	5'129928
136	18496	2515456	11'6619038	5'142563
137	18769	2571353	11'7046999	5'155137
138	19044	2628072	11'7473444	5'167649
139	19321	2685619	11'7898261	5'180101
140	19600	2744000	11'8321596	5'192494
141	19881	2803221	11'8743421	5'204828
142	20164	2863288	11'9163753	5'217103
143	20449	2924207	11'9582607	5'229321
144	20736	2985984	12'0000000	5'241482
145	21025	3048625	12'0415946	5'253588
146	21316	3112136	12'0830460	5'265637
147	21609	3176523	12'1243557	5'277632
148	21904	3241792	12'1655251	5'289572
149	22201	3307949	12'2065556	5'301459
150	22500	3375000	12'2474487	5'313293
151	22801	3442951	12'2882057	5'325074
152	23104	3511808	12'3288280	5'336803
153	23409	3581577	12'3693169	5'348481
154	23716	3652264	12'4096736	5'360108
155	24025	3723875	12'4498996	5'371685
156	24336	3796416	12'4899960	5'383213
157	24649	3867893	12'5299641	5'394690
158	24964	3944312	12'5698051	5'406120
159	25281	4019619	12'6095202	5'417501
160	25600	4096000	12'6491106	5'428835
161	25921	4173281	12'6885775	5'440122
162	26244	4251528	12'7279221	5'451362
163	26569	4330747	12'7671453	5'462556
164	26896	4410944	12'8062485	5'473703
165	27225	4492125	12'8452326	5'484805

Num- ber.	Square.	Cube.	Square Root.	Cube Root.
166	27556	4574296	12.8840987	5.495865
167	27889	4657463	12.9228480	5.506879
168	28224	4741632	12.9614814	5.517848
169	28561	4826809	13.0000000	5.528775
170	28900	4913000	13.0384048	5.539658
171	29241	5000211	13.0766968	5.550499
172	29584	5088448	13.1148770	5.561298
173	29929	5177717	13.1529464	5.572054
174	30276	5268024	13.1909060	5.582770
175	30625	5359375	13.2287566	5.593445
176	30976	5451776	13.2664992	5.604079
177	31329	5545233	13.3041347	5.614673
178	31684	5639752	13.3416641	5.625226
179	32041	5735339	13.3790882	5.635741
180	32400	5832000	13.4164079	5.646216
181	32761	5929741	13.4536240	5.656552
182	33124	6028568	13.4907376	5.667051
183	33489	6128487	13.5277493	5.677411
184	33856	6229504	13.5646600	5.687734
185	34225	6331625	13.6014705	5.698019
186	34596	6434856	13.6381817	5.708267
187	34969	6539203	13.6747943	5.718479
188	35344	6644672	13.7113092	5.728654
189	35721	6751269	13.7477271	5.738794
190	36100	6859000	13.7840488	5.748897
191	36481	6967871	13.8202750	5.758965
192	36864	7077888	13.8564065	5.768998
193	37249	7189057	13.8924440	5.778996
194	37636	7301384	13.9283883	5.788660
195	38025	7414875	13.9642400	5.798890
196	38416	7529536	14.0000000	5.808786
197	38809	7645373	14.0356688	5.818648
198	39204	7762392	14.0712473	5.828476
199	39601	7880599	14.1067360	5.838272
200	40000	8000000	14.1421356	5.848035

M E N S U R A T I O N.

MENSURATION is the measuring and estimating the magnitude and dimensions of bodies and figures: and it is either angular, lineal, superficial, or solid, according to the objects it is concerned with. It is accordingly treated in several parts: as 1st, Practical Geometry, which treats of the definitions and construction of geometrical figures; 2^d, Trigonometry, which teaches the calculation and construction of triangles, or three-sided figures, and, by application, of other figures depending on them: 3^d, Superficial Mensuration, or the measuring the surfaces of bodies; 4th, Solid Mensuration, or measuring the capacities or solid contents of bodies. Beside these general heads, there are several other subordinate divisions, as also the application of them to the practical concerns of life. Of each of which in their order: excepting Trigonometry, which is fully treated of in my large book on Mensuration, as also in my New Course of Mathematics.

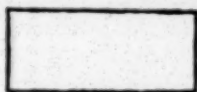
P R A C T I C A L G E O M E T R Y.

D E F I N I T I O N S.

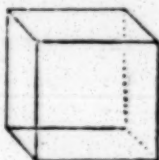
1. **A** POINT has position, but no parts nor dimensions, neither length, breadth, nor thickness.

2. A line is length, without breadth or thickness.

3. A surface or superficies, is an extension, or a figure, of two dimensions, length and breadth ; but without thickness.



4. A body or solid, is a figure of three dimensions, namely, length, breadth, and thickness.



Hence surfaces are the extremities of solids ; lines the extremities of surfaces ; and points the extremities of lines.

5. Lines are either right, or curved, or mixed of these two.



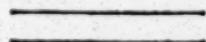
6. A right line, or straight line, lies all in the same direction, between its extremities ; and is the shortest distance between two points.



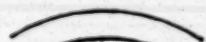
7. A curve continually changes its direction between its extreme points.



8. Lines are either parallel, oblique, perpendicular, or tangential.



9. Parallel lines are always at the same distance ; and never meet though ever so far produced.



10. Oblique right lines change their distance, and would meet, if produced, on the side of the least distance.



11. One line is perpendicular to another, when it inclines not more on the one side than on the other.



12. One line is tangential, or a tangent to another, when it touches it, without cutting, when both are produced.



13. An angle is the inclination, or opening, of two lines, having different directions, and meeting in a point.



14. Angles are right or oblique, acute or obtuse.

15. A right angle, is that which is made by one line perpendicular to another. Or when the angles on each side are equal to one another, they are right angles.



16. An oblique angle, is that which is made of two oblique lines; and is either less or greater than a right angle.



17. An acute angle is less than a right angle.

18. An obtuse angle is greater than a right angle.



19. Surfaces are either plane or curved.

20. A plane, or plane superficies, is that with which a right line may, every way, coincide. But if not it is curved.

21. Plane figures are bounded either by right lines or curves.

22. Plane figures that are bounded by right lines, have names according to the number of their sides, or of their angles; for they have as many sides as angles; the least number being three.

23. A figure of three sides and angles, is called a triangle. And it receives particular denominations from the relations of its sides and angles.

24. An equilateral triangle, is that whose three sides are all equal.



25. An

25. An isosceles triangle, is that which has two sides equal.



26. A scalene triangle, is that whose three sides are all unequal.



27. A right-angled triangle, is that which has one right angle.



28. Other triangles are oblique-angled, and are either obtuse or acute.

29. An obtuse-angled triangle has one obtuse angle.



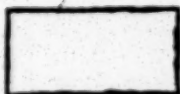
30. An acute-angled triangle has all its three angles acute.



31. A figure of four sides and angles, is called a quadrangle, or a quadrilateral.

32. A parallelogram is a quadrilateral which has both its pairs of opposite sides parallel. And it takes the following particular names.

33. A rectangle is a parallelogram, having all its angles right ones.



34. A square is an equilateral rectangle; having all its sides equal, and all its angles right ones.



35. A rhomboid is an oblique-angled parallelogram.



36. A rhombus is an equilateral rhomboid; having all its sides equal, but its angles oblique.



37. A trapezium is a quadrilateral which hath not both its pairs of opposite sides parallel.



38. A trapezoid hath only one pair of opposite sides parallel.



39. A diagonal is a right line joining any two opposite angles of a quadrilateral.



40. Plane figures that have more than four sides are, in general, called polygons: and they receive other particular names according to the number of their sides or angles.

41. A pentagon is a polygon of five sides; a hexagon hath six sides; a heptagon, seven; an octagon, eight; a nonagon, nine; a decagon, ten; an undecagon, eleven; and a dodecagon hath twelve sides.

42. A regular polygon hath all its sides and all its angles equal. If they are not both equal, the polygon is irregular.

43. An equilateral triangle is also a regular figure of three sides, and the square is one of four: the former being also called a trigon, and the latter a tetragon.

44. A circle is a plane figure bounded by a curve line, called the circumference, which is every where equi distant from a certain point within, called its centre.



Note, The circumference itself is often called a circle.

45. The

45. The radius of a circle, is a right line drawn from the centre to the circumference.



46. The diameter of a circle, is a right line drawn through the centre, and terminating in the circumference on both sides.



47. An arc of a circle, is any part of the circumference.



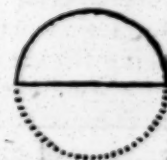
48. A chord, is a right line joining the extremities of an arc.



49. A segment, is any part of a circle, bounded by an arc and its chord.



50. A semicircle, is half the circle, or a segment cut off by a diameter.



51. A sector, is any part of a circle, bounded by an arc, and two radii drawn to its extremities.



52. A quadrant, or quarter of a circle, is a sector having a quarter of the circumference for its arc, and its two radii are perpendicular to each other.

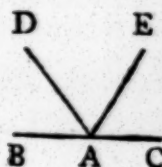


53. The height or altitude of a figure, is a perpendicular let fall from an angle, or its vertex, to the opposite side, called the base.



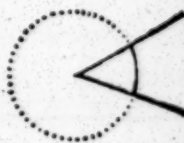
54. In a right-angled triangle, the side opposite the right angle, is called the hypotenuse; and the other two sides, the legs, or sometimes the base and perpendicular.

55. When an angle is denoted by three letters, of which one stands at the angular point, and the other two on the two sides, that which stands at the angular point is read in the middle.



56. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; and each degree into 60 minutes, each minute into 60 seconds, and so on. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

57. The measure of a right-lined angle, is an arc of any circle contained between the two lines which form that angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc. Hence a right angle is an angle of 90 degrees.



The definition of solids, or bodies, will be given afterwards, when we come to treat of the mensuration of solids.

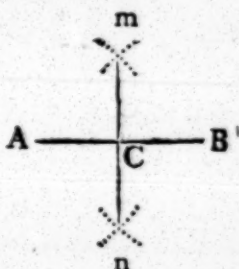
PROBLEMS,

P R O B L E M S.

P R O B L E M I.

To divide a Given Line AB into Two Equal Parts.

From the centres A and B, with any distance greater than half AB. describe arcs cutting each other in m and n. Draw the line mCn, and it will cut the given line into two equal parts in the middle point C.



P R O B L E M II.

To divide a Given Angle ABC into Two Equal Parts.

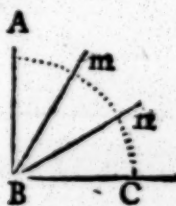
From the centre B, with any distance, describe the arc AC. From A and C, with one and the same radius, describe arcs intersecting in m. Draw the line Bm, and it will bisect the angle as required.



P R O B L E M III.

To divide a Right Angle ABC into Three Equal Parts.

From the centre B, with any distance, describe the arc AC. From the centre A, with the same radius, cross the arc AC in n. And with the centre C, and the same radius, cut the arc AC in m. Then through the points m and n draw Bm and Bn, and they will trisect the right angle as required.



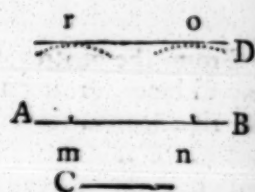
P R O.

PROBLEM IV.

To draw a Line Parallel to a Given Line AB.

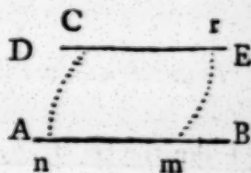
CASE 1. *When the Parallel Line is to be at a Given Distance C.*

From any two points m and n , in the line AB , with a distance equal to C , describe the arcs r and o .—Draw CD to touch these arcs, without cutting them, and it will be the parallel required.



CASE 2. *When the Parallel Line is to pass through a Given Point C.*

From any point m , in the line AB , with the distance mC , describe the arc Cn .—From the centre C with the same radius, describe the arc mr . Take the arc Cn in the compasses, and apply it from m to r .—Through C and r draw DE , the parallel required.



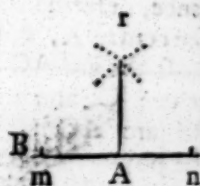
Note. This problem is more easily effected with a parallel ruler.

PROBLEM V.

To erect a Perpendicular from a Given Point A in a Given Line BC.

CASE 1. *When the Point is near the Middle of the Line.*

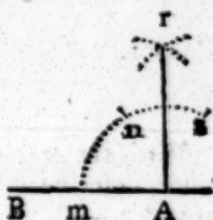
On each side of the point A take any two equal distances Am , An . From the centres m , n , with any radius greater than Am or An , describe two arcs cutting in r .—Through A and r draw the line Ar , and it will be the perpendicular as required.



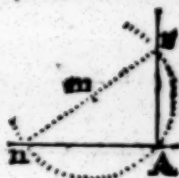
CASE

CASE 2. *When the Point is near the End of the Line.*

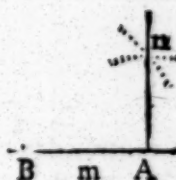
With the centre A, and any distance describe the arc mns .—From the point m , with the same radius, turn the compasses twice over on the arc, as at n and s .—Again, with the centres n and s , describe arcs intersecting in r .—Then draw Ar , and it will be the perpendicular as required.

*Another Method.*

From any point m as a centre, with the radius or distance mA , describe an arc cutting the given line in n and A .—Through n and m draw a right line cutting the arc in r .—Lastly, draw Ar , and it will be the perpendicular as required.

*Another Method.*

From any plane scale of equal parts, set off Am equal to 4 parts. —With centre A, and distance of 3 parts, describe an arc—And with centre m , and radius of 5 parts, cross it at n .—Draw An for the perpendicular required.



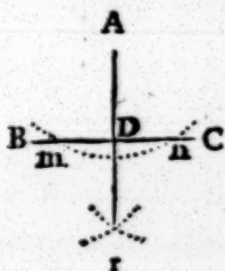
Or any other numbers in the same proportion, as 3, 4, 5, will do the same; such as 6, 8, 10, &c.

PROBLEM VI.

From a Given Point A, out of a Given Line BC, to let fall a Perpendicular.

CASE 1. *When the Point is nearly opposite the Middle of the Line.*

With the centre A, and any distance, describe an arc cutting BC in m and n. — With the centres m and n, and the same, or any other radius, describe arcs intersecting in r. — Draw ADr, for the perpendicular required.

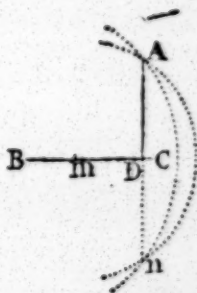


CASE 2. *When the Point is nearly opposite the End of the Line.*

From A draw any line Am to meet BC, in any point m. — Bisect Am at n, and with the centre n, and distance An or mn, describe an arc, cutting BC in D. — Draw AD the perpendicular as required.

*Another Method.*

From B, or any point in BC, as a centre, describe an arc through the point A. — From any other centre m in BC, describe another arc through A, and cutting the former arc again in n. — Through A and n draw the line ADn; and AD will be the perpendicular as required.



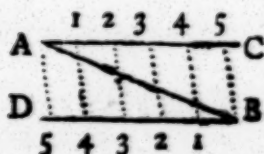
Note.

Note. Perpendiculars may be more readily raised and let fall, in practice, by means of a square, or by the common parallelogram protractor.

PROBLEM VII.

To divide a Given Line AB into any proposed Number of Equal Parts.

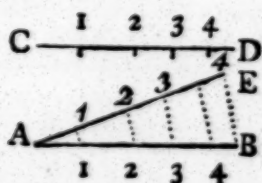
From A draw any line AC at random, and from B draw BD parallel to it.—On each of these lines, beginning at A and B, set off as many equal parts of any length, as AB is to be divided into. Join the opposite points of division by the lines A 5, 1 4, 2 3, &c; and they will divide the given line AB as required.



PROBLEM VIII.

To divide a Given Line AB in the same Proportion as another Line CD is divided.

From A draw any line AE equal to CD, and upon it transfer the divisions of the line CD. —Join BE, and parallel to it draw the lines 1 1, 2 2, 3 3, &c; and they will divide the line AB as required.

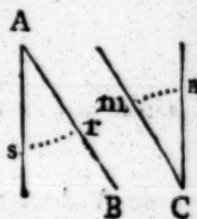


PROV

PROBLEM IX.

At a Given Point A, in a Given Line AB, to make an Angle Equal to a Given Angle C.

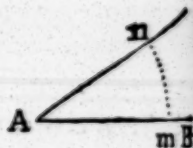
With the centre C, and any distance, describe an arc mn.—With the centre A, and the same radius, describe the arcs rs.—Take the distance mn between the compasses, and apply it from r to s.—Then a line drawn through A and s, will make the angle A equal to the angle C as required.



PROBLEM X.

At a Given Point A, in a Given Line AB, to make an Angle of any proposed Number of Degrees.

With the centre A, and radius equal to 60 degrees, taken from a scale of chords, describe an arc, cutting AB in m.—Then take between the compasses, the proposed number of degrees from the same scale of chords, and apply them from m to n. Through the point n draw An, and it will make the angle A of the number of degrees proposed.



Note. Angles of more than 90 degrees are usually taken off at twice.

Or the angle may be made with the protractor or other instrument, by laying the centre to the point A, and its radius along AB; then make a mark n at the proposed number of degrees, through which draw the line An as before.

PRO-

PROBLEM XI.

*To measure a Given Angle A.**(See the last Figure.)*

Describe the arc mn with the chord of 60 degrees, as in the last Problem.—Take the arc mn between the compasses, and that extent, applied to the chords, will shew the degrees in the given angle.

Note. When the distance mn exceeds 90 degrees, it must be taken off at twice, as before.

Or the angle may be measured by applying the radius of a graduated arc, of any instrument, to AB , as in the last problem; and then noting the degrees cut off by the other leg Ac of the angle.

PROBLEM XII.

To find the Centre of a Circle.

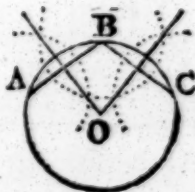
Draw any chord AB ; and bisect it perpendicularly with CD , which will be a diameter. Bisect CD in the point O ; and that will be the centre,



PROBLEM XIII.

To describe the Circumference of a Circle through Three Given Points A, B, C.

From the middle point B draw chords to the two other points.—Bisect the chords perpendicularly by lines meeting in O , which will be the centre.—Then from the centre O , at the distance OA , or OB , or OC , describe the circle.

*Note.*

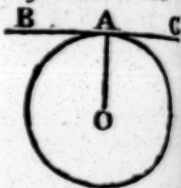
Note. In the same manner may the centre of an arc of a circle be found.

PROBLEM XIV.

Through a Given Point A, to draw a Tangent to a Given Circle.

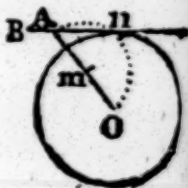
CASE 1. *When A is in the Circumference of the Circle.*

From the given point A, draw AO to the centre of the circle.—Then through A draw BC perpendicular to AO, and it will be the tangent as required.



CASE 2. *When A is out of the Circumference.*

From the given point A, draw AO to the centre, which bisect in the point m.—With the centre m, and radius mA or mO, describe an arc cutting the given circle in n.—Through the points A and n, draw the tangent BC.

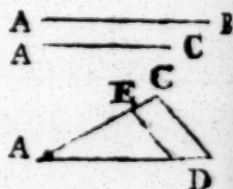


PROBLEM XV.

To find a Third Proportional to Two Given Lines, AB, AC.

Place the two given lines, AB, AC, making any angle at A, and join BC.—In AB take AD equal to AC, and draw DE parallel to BC. So shall AE be the third proportional to AB and AC.

That is, $AB : AC :: AC : AE$.

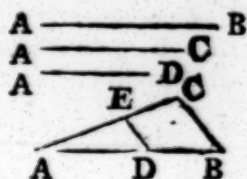


PROBLEM XVI.

To find a Fourth Proportional to Three Given Lines, AB, AC, AD.

Place two of them AB, AC, making any angle at A, and join BC. Place AD on AB, and draw DE parallel to BC. So shall AE be the fourth proportional required.

That is, $AB : AC :: AD : AE$.

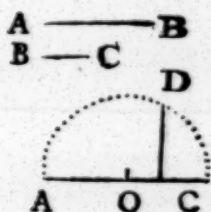


PROBLEM XVII.

To find a Mean Proportional between Two Given Lines AB, BC.

Join AB and BC in one straight line AC, and bisect it in the point O.—With the centre O, and radius OA or OC, describe a semicircle.—Erect the perpendicular BD, and it will be the mean proportional required.

That is, $AB : BD :: BD : BC$.



PROBLEM XVIII.

To make an Equilateral Triangle on a Given Line AB.

From the centres A and B, with the distances AB, describe arcs, intersecting in C.—Draw AC and BC, and it is done.

Note. An isosceles triangle may be made in the same manner, taking for the distance the given length of one of the equal sides.

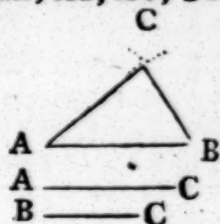


PRO-

PROBLEM XIX.

To make a Triangle with Three Given Lines, AB, AC, BC.

With the centre A and distance AC, describe an arc.—With the centre B, and distance BC, describe another arc, cutting the former in C.—Draw AC and BC, and ABC is the triangle required.



PROBLEM X

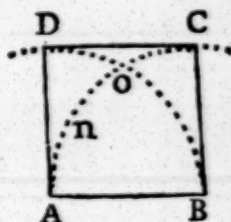
To make a Square on a Given Line AB.

Draw BC perpendicular and equal to AB, From A and C, with the distance AB, describe arcs intersecting in D.—Draw AD and CD, and it is done.



Another Way.

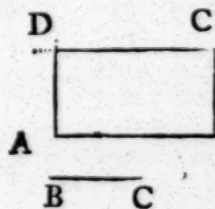
On the centres A and B, with the distance AB, describe arcs crossing at o.—Bisect Ao in n.—With centre o, and radius on, cross the two arcs in C and D.—Then draw AD, BC, CD.



PROBLEM XXI.

To describe a Rectangle, or a Parallelogram, of a Given Length and Breadth.

Place BC perpendicular to AB.—With centre A, and distance AC, describe an arc.—With centre C, and radius AB, describe another arc, cutting the former in D.—Draw AD and CD, and it is done.



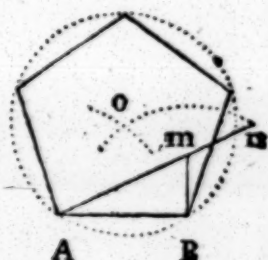
Note.

Note. In the same manner is described any oblique parallelogram, only drawing BC, to make the given oblique angle with AB, instead of perpendicular to it.

PROBLEM XXII.

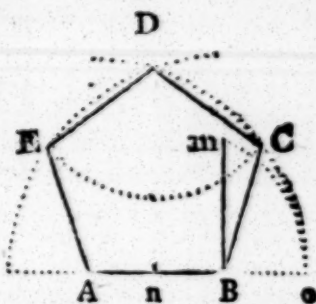
To make a Regular Pentagon on a Given Line AB.

Make Bm perpendicular and equal to half AB.—Draw Am, and produce it till mn be equal to Bm.—With centres A and B, and distance Bn, describe arcs intersecting in o, which will be the centre of the circumscribing circle.—Then with the centre o, and the same radius, describe the circle; and about the circumference of it apply AB the proper number of times.



Another Method.

Make Bm perpendicular and equal to AB.—Bisect AB in n; then with the centre n, and distance nm, cross AB produced in o.—With the centres A and B, and distance Ao, describe arcs intersecting in D, which will be the opposite angle of the pentagon.—Lastly, with centre D, and radius AB, cross those arcs again in C and E, the other two angles of the figure.—Then draw the lines from angle to angle, to complete the figure.

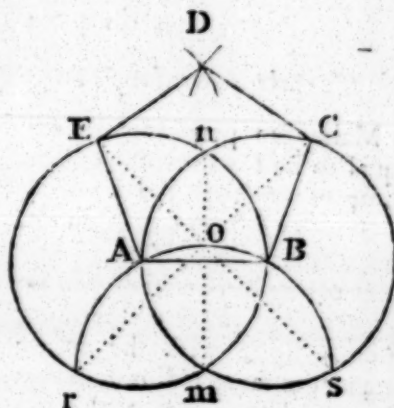


E

A Third

A Third Method, nearly true.

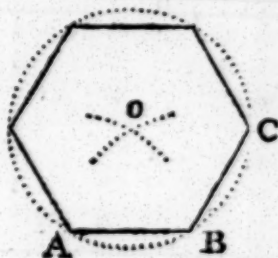
On the centres A and B, with the distance AB, describe two circles intersecting in m and n. — With the same radius, and the centre m, describe rAoBS, and draw mn cutting it in o. — Draw roC and SoE, which will give two angles of the pentagon. — Lastly, with radius AB, and centres C and E, describe arcs intersecting in D, which will be the other angle of the pentagon nearly.



PROBLEM XXIII.

To make a Hexagon on a Given Line AB.

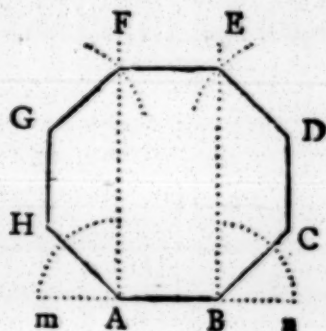
With the distance AB, and the centres A and B, describe arcs intersecting in o. — With the same radius, and centre o, describe a circle, which will circumscribe the hexagon. — Then apply the line AB six times round the circumference, marking out the angular points; and connect them with right lines.



PROBLEM XXIV.

To make an Octagon on a Given Line AB.

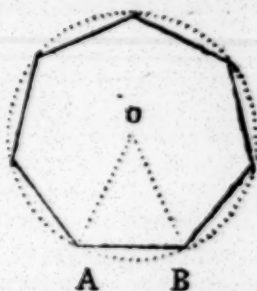
Erect AF and BE perpendicular to AB. — Produce AB both ways, and bisect the angles mAF and nBE with the lines AH and BC, each equal to AB. — Draw CD and HG parallel to AF or BE, and each equal to AB. — With the distance AB, and centres G and D, cross AF and BE in F and E. — Then join GF, FE, ED, and it is done.



PROBLEM XXV.

To make any Regular Polygon on a Given Line AB.

Draw Ao and Bo making the angles A and B each equal to half the angle of the polygon. — With the centre o and distance oA, describe a circle. — Then apply the line AB continually round the circumference the proper number of times, and it is done.



Note. The angle of any polygon, of which the angles oAB and oBA are each one half, is found thus: Divide the whole 360 degrees by the number of sides,

sides, and the quotient will be the angle at the centre o ; then subtract that from 180 degrees, and the remainder will be the angle of the polygon, and is double of oAB or of oBA . And thus you will find the numbers of the following table, containing the degrees in the angle o , at the centre, and the angle of the polygon, for all the regular figures from 3 to 12 sides.

No. of sides	Name of the Polygon	Angle o at the centre	Angle of the polyg.	Angle oAB or oBA
3	Trigon	120°	60°	30°
4	Tetragon	90	90	45
5	Pentagon	72	108	54
6	Hexagon	60	120	60
7	Heptagon	51 $\frac{3}{7}$	128 $\frac{4}{7}$	64 $\frac{2}{7}$
8	Octagon	45	135	67
9	Nonagon	40	140	70
10	Decagon	36	144	72
11	Undecagon	32 $\frac{8}{11}$	147 $\frac{3}{11}$	73 $\frac{7}{11}$
12	Dodedagon	30	150	75

PROBLEM XXVI.

In a Given Circle to inscribe any Regular Polygon; or to divide the Circumference into any Number of Equal Parts.

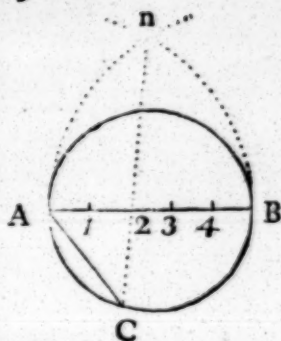
(See the last figure.)

At the centre o make an angle equal to the angle at the centre of the polygon, as contained in the third column of the above table of polygons.—Then the distance AB will be one side of the polygon; which being carried round the circumference the proper number of times, will complete the figure.—Or, the arc AB will be one of the equal parts of the circumference.

Another

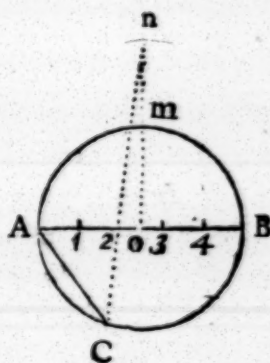
Another Method, nearly true.

Draw the diameter AB, which divide into as many equal parts as the figure has sides.—With the distance AB, and centres A and B, describe arcs crossing at n: from thence draw nC through the second division on the diameter; so shall AC be a side of the polygon, nearly.



Another Method, still nearer.

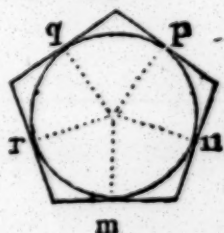
Divide the diameter AB, as before, into as many equal parts as the figure has sides. From the centre o raise the perpendicular om, which produce till mn be equal to three-fourths, of the radius om.—From n draw nC through the second division of the diameter, and the line AC will be the side of the polygon still nearer than before; or the arc AC one of the equal parts into which the circumference is to be divided.



PROBLEM XXVII.

About a Given Circle to Circumscribe any Polygon.

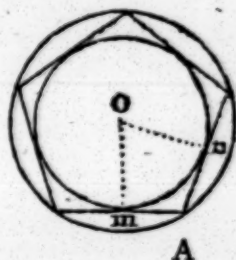
Find the points m, n, p, &c. as in the last problem; to which draw radii mo, no, &c. to the centre of the circle.—Then through these points m, n, &c. and perpendicular to these radii, draw the sides of the polygon.



PROBLEM XXVIII.

To find the Centre of a Given Polygon, or the centre of its Inscribed or Circumscribe Circle.

Bisect any two sides with the perpendiculars mo , no ; and their intersection will be the centre.—Then with the centre o , and the distance om , describe the inscribed circle; or with the distance to one of the angles, as A , describe the circumscribing circle.

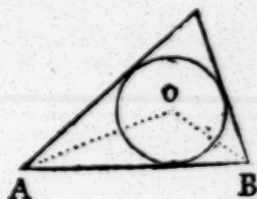


Note. This method will also circumscribe a circle about any given oblique triangle.

PROBLEM XXIX.

In any Given Triangle to Inscribe a Circle.

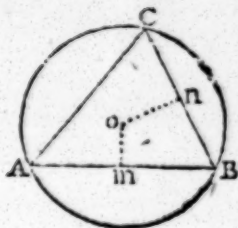
Bisect any two of the angles with the lines Ao , Bo ; and o will be the centre of the circle.—Then with the centre o , and radius the nearest distance to any one of the sides, describe the circle.



PROBLEM XXX.

About any Given Triangle to Circumscribe a Circle.

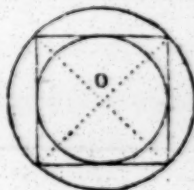
Bisect any two of the sides AB, BC, with the perpendiculars mo, no, — With the centre o, and distance to any one of the angles, describe the circle.



PROBLEM XXXI.

In, or About, a Given Square, to describe a Circle.

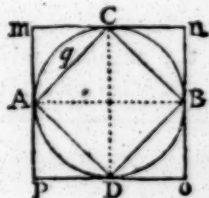
Draw the two diagonals of the square, and their intersection o will be the centre of both the circles.—Then with that centre, and the nearest distance to one side, describe the inner circle; and with the distance to one angle, describe the outer circle.



PROBLEM XXXII.

In, or About, a Given Circle, to describe a Square, or an Octagon.

Draw two diameters AB, CD, perpendicular to each other.—Then connect their extremities, and that will give the inscribed square ACBD.—Also through their extremities draw tangents parallel to them, and they will form the outer square mnop.

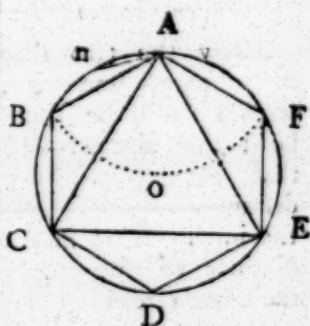


Note. If any quadrant, as AC, be bisected in q , it will give one-eighth of the circumference, or the side of the octagon.

PROBLEM XXXIII.

In a Given Circle, to Inscribe a Trigon, a Hexagon, or a Dodecagon.

The radius of the circle is the side of the hexagon. Therefore from any point A in the circumference, with the distance of the radius, describe the arc BOF. Then is AB the side of the hexagon; and therefore carrying it six times round will form the hexagon, or will divide the circumference into six equal parts, each containing 60 degrees—The second of these C, will give AC the side of the trigon, or equilateral triangle ACE, and the arc AC one-third of the circumference, or 120 degrees.—Also the half of AB, or An is one-12th of the circumference, or 30 degrees, which gives the side of the dodecagon.

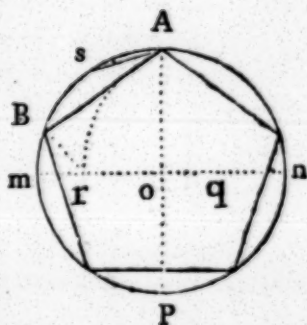


Note. If tangents to the circle be drawn through all the angular points of any inscribed figure, they will form the sides of a like circumscribing figure.

PROBLEM XXXIV.

In a Given Circle to Inscribe a Pentagon, or a Decagon.

Draw the two diameters AP, mn perpendicular to each other, and bisect the radius on at q.—With the centre q and distance qA, describe the arc Ar; and with the centre A, and radius Ar, describe the arc rB. Then is AB one-fifth of the circumference: and AB carried five times over will form the pentagon. Also the arc AB bisected in s, will give As the tenth part of the circumference, or the side of the decagon.



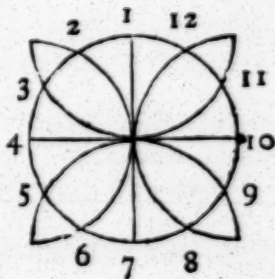
Note. Tangents being drawn through the angular points, will form the circumscribing pentagon or decagon.

PROBLEM XXXV.

To divide the Circumference of a Given Circle into 12 Equal Parts, each of 30 Degrees.

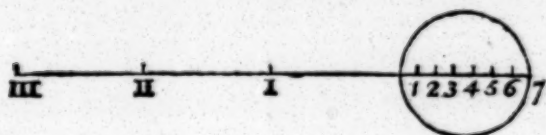
Or to Inscribe a Dodecagon by another Method.

Draw two diameters 1 7 and 4 10 perpendicular to each other.—Then with the radius of the circle, and the four extremities, 1, 4, 7, 10, as centres, describe arcs, through the centre of the circle; and they will cut the circumference in the points required, dividing it into 12 equal parts, at the points marked with the numbers in the figure.



PROBLEM XXXVI.

To draw a Right Line equal to the Circumference of a Given Circle.

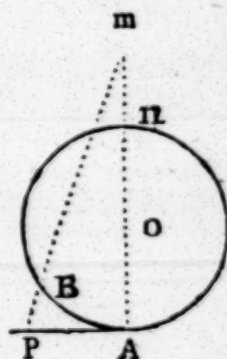


Take III I equal to 3 times the diameter and $\frac{1}{7}$ part more: and it will be equal to the circumference, very nearly.

PROBLEM XXXVII.

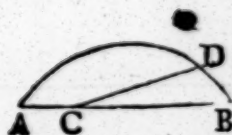
To find a Right Line equal to any Given Arc AB of a Circle.

Through the point A and the centre draw Am, making mn equal to $\frac{3}{4}$ of the radius no.—Also draw the indefinite tangent AP perpendicular to it.—Then through m and B draw mB: so shall AP be equal to the arc AB very nearly.



Otherwise.

Divide the chord AB into 4 equal parts.—Set one part AC on the arc from B to D.—Draw CD, and the double of it will be nearly equal to the arc ADB.



PRO-

PROBLEM XXXVIII.

To divide a Given Circle into any proposed Number of Parts by Equal Lines, so that these Parts shall be mutually Equal both in Area and Perimeter.

Divide the diameter AB into the proposed number of equal parts at the points a, b, c, &c.—Then on Aa, Ab, Ac, &c. as diameters, describe semicircles on one side of the diameter AB; and on Bd Bc, Bb, &c. describe semicircles on the other side of the diameter. So shall the corresponding joining semicircles divide the given circle in the manner proposed.



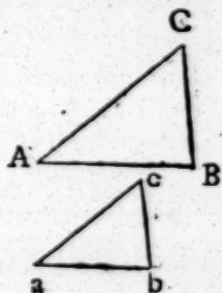
And in like manner we may proceed when the spaces are to be in any given proportion.—As to the perimeters, they are always equal, whatever be the proportion of the spaces.

PROBLEM XXXIX.

To make a Triangle Similar to a Given Triangle ABC.

Let AB be one side of the required Triangle. Make the angle a equal to the angle A, and the angle b equal to the angle B; then the triangle abc will be similar to ABC as proposed.

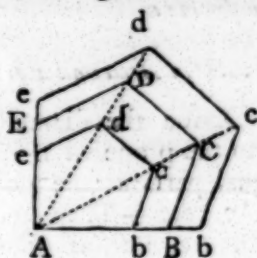
Note. If ab be equal to AB, the triangles will also be equal, as well as similar.



PROBLEM XL.

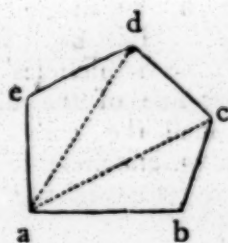
To make a Figure similar to any other Given Figure ABCDE.

From any angle A draw diagonals to the other angles. —Take Ab a side of the figure required. Then draw bc parallel to BC, and ed to CD, and de to DE, &c.



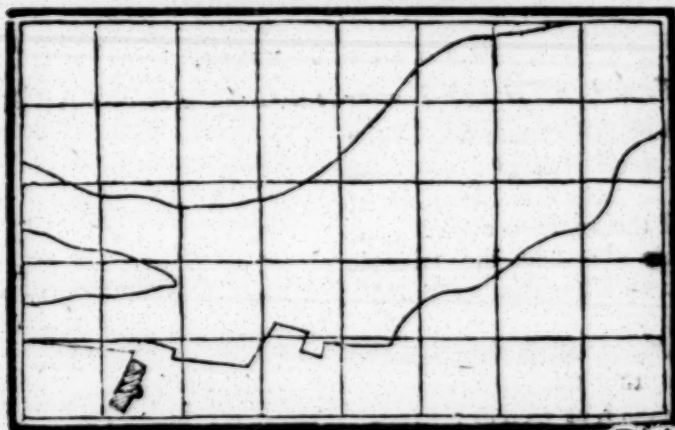
Otherwise.

Make the angles at a, b, e, respectively equal to the angles at A, B, E, and the lines will intersect in the corners of the figure required.

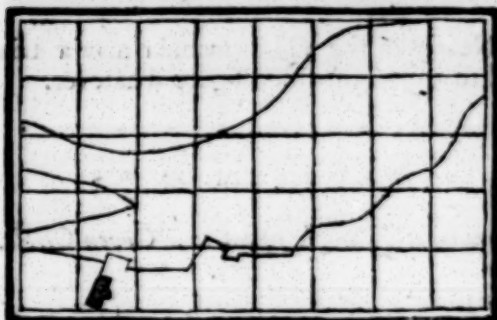


PROBLEM XLI.

To reduce a Complex Figure from one Scale to another, mechanically, by means of Squares.



Divide

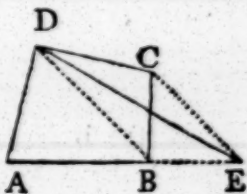


Divide the given figure, by cross lines, into squares, as small as may be thought necessary.—Then divide another paper into the same number of squares, and either greater, equal or less, in the given proportion.—This done, observe what squares the several parts of the given figure are in, and draw with a pencil, similar parts in the corresponding squares of the new figure. And so proceed till the whole is copied.

PROBLEM XLII.

To make a Triangle equal to a Given Trapezium ABCD.

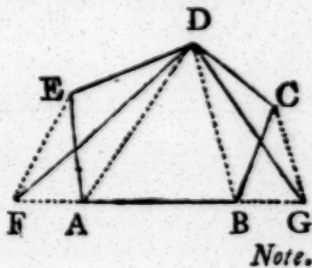
Draw the diagonal DB, also CE parallel to it, meeting AB produced in E.—Join DE; so shall the triangle ADE be equal to the trapezium ABCD.



PROBLEM XLIII.

To make a Triangle equal to the Figure ABCDEA.

Draw the diagonals DA, DB, and the lines EF, CG parallel to them, meeting the base AB, both ways produced, in F and G.—Join DF, DG; and DFG will be the triangle required, equal to the given figure ABCDE.



Note.

Note. Nearly in the same manner may a triangle be made equal to any right-lined figure whatever.

PROBLEM XLIV.

To make a Triangle equal to a Given Circle.

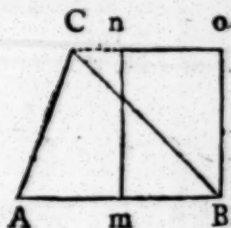
Draw any radius AO , and the tangent AB perpendicular to it. — On which take B AB equal to the circumference of the circle by Problem xxxvi. — Join BO ; so shall ABO be the triangle required, equal to the given circle, nearly.



PROBLEM XLV.

To make a Rectangle, or a Parallelogram, equal to a Given Triangle ABC.

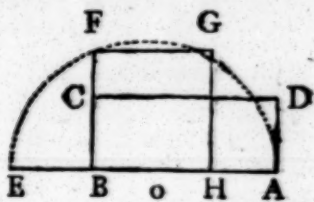
Bisect the base AB in m . — Through C draw Cno parallel to AB . — Through m and B draw mn and BO parallel to each other, and either perpendicular to AB , or making any angle with it. And the rectangle or parallelogram $mnoB$ will be equal to the triangle, as required.



PROBLEM XLVI.

To make a Square equal to a Given Rectangle ABCD.

Produce one side, AB, till BE be equal to the other side BC.—Bisect AE in o; on which as a centre, with radius Ao, describe a semi-circle, and produce BC to meet it at F.—On BF make the square BFGH, and it will be equal to the rectangle ABCD, as required.

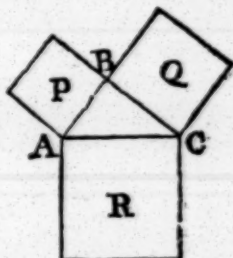


∴ Thus the circle, and all right-lined figures, have been reduced to equivalent squares.

PROBLEM XLVII.

To make a Square equal to Two Given Squares P and Q.

Set two sides AB, BC, of the given squares, perpendicular to each other.—Join their extremities AC; so shall the square R, constructed on AC, be equal to the two P and Q taken together.



Note. Circles, or any other similar figures, are added in the same manner. For, if AB and BC be the diameters of two circles, AC will be the diameter of a third circle equal to both the other two. And if AB and BC be the like sides of any two similar figures, then AC will be the like side of another similar figure equal to both the two former, and on which the third figure may be constructed by Problem XL.

P R O-

PROBLEM XLVIII.

To make a Square equal to the Difference between Two Given Squares P, R.

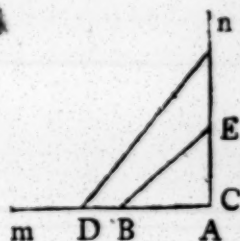
(See the last figure.)

On the side AC of the greater square, as a diameter, describe a semicircle; in which apply AB the side of the less square.—Join BC, and it will be the side of a square equal to the difference between the two P and R, as required.

PROBLEM XLIX.

To make a Square equal to the Sum of any Number of Squares taken together.

Draw two indefinite lines Am, An, perpendicular to each other at the point A. On the one of these set off AB the side of one of the given squares, and on the other AC the side of another of them. Join BC, and it will be the side of a square equal to the two together.



Then take AD equal to BC, and AE equal to the side of the third given square. So shall DE be the side of a square equal to the sum of the three given squares.—And so on continually, always setting more sides of the given squares on the line An, and the sides of the successive sums on the other line Am.

Note. And thus any number of any sort of figures may be added together.

P R O-

PROBLEM L.

To make Plane Diagonal Scales.



Draw any line as AB, of any convenient length. Divide it into 11 equal parts.* Complete these into rectangles of a convenient height, by drawing parallel and perpendicular lines. Divide the altitude into 10 equal parts, if it be for a decimal scale for common numbers, or into 12 equal parts, if it be for feet and inches; and through these points of division draw as many parallel lines, the whole length of the scale.—Then divide the length of the first division AC into 10 equal parts, both above and below; and connect these points of division by diagonal lines, and the scale is finished, after being numbered as you please.

Note. These diagonal scales serve to take off divisions or numbers of three figures. If the first large divisions be units; the second set of divisions along AC, will be 10th parts; and the divisions in the altitude, along AD will be 100th parts. If CD be tens, AC will be units, and AD will be 10th parts. If CB be hundreds, AC will be tens, and AD units. If CB be thousands, AC will be hundreds, and AD will be tens. And so on, each set of divisions being tenth parts of the former ones.

For example, suppose it were required to take off 243 from the scale. Fix one foot of the comp. at 2 of the greatest divisions, at the bottom of the scale, and

* Only 4 parts are here drawn for want of room.

extend

extend the other to 4 of the second divisions, along the bottom; then, for the three, slide up both points of the compasses by a parallel motion, till they fall upon the third longitudinal line; and in that position extend the second point of the compasses to the fourth diagonal line, and you have the extent of three figures as required.

Or, if you have any line to measure the length of.—Take it between the compasses, and applying it to the scale, suppose it fall between 3 and 4 of the large divisions: or, more nearly, that it is 3 of the large divisions, or 3 hundreds, and between 5 and 6 of the second divisions, or 5 tens or 50, and a little more. Slide up the points of the compasses by a parallel motion, keeping one foot always on the vertical division of 3 hundred, till the other point fall exactly on one of the diagonal lines, which suppose to be 8, being 8 units, Which shews that the length of the line, proposed to be measured, is 358.

PLANE SCALES FOR TWO FIGURES.



The above are three other forms of scales, the first of which is a decimal scale, for taking off common numbers consisting of two figures. The other two are duodecimal scales, and serve for feet and inches, &c.

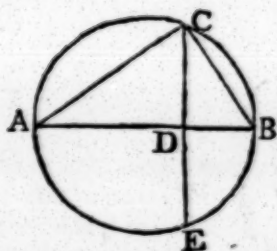
3

These,

These, and other scales, engraved on ivory, are fittest for practical use. And the most convenient form of a plane scale of equal divisions, is on the very edge of the ivory, made thin at the edge for laying along any line, and then marking the paper opposite any division required: which is better than taking lengths off a scale with compasses.

R E M A R K S.

Note 1. That in a circle, the half chord DC, is a mean proportional between the segments AD, DB of the diameter AB perpendicular to it. That is $AD : DC :: DC : DB$.



2. The chord AC is a mean proportional between AD and the diameter AB. And the chord BC a mean proportional between DB and AB.

That is, $AD : AC :: AC : AB$,
and $BD : BC :: BC : AB$.

3. The angle ACB, in a semicircle, is always a right angle.

4. The square of the hypotenuse of a right-angled triangle, is equal to the square of both the sides.

That is, $AC^2 = AD^2 + CD^2$,
and $BC^2 = BD^2 + DC^2$,
and $AB^2 = AC^2 + BC^2$.

5. Triangles that have all the three angles of the one respectively equal to all the three of the other, are called equiangular triangles, or similar triangles.

6. In similar triangles, the like sides, or sides opposite the equal angles, are proportional.

7. The areas, or spaces, of similar triangles, are to each other, as the squares of their like sides.

M E N

M E N S U R A T I O N

O F

S U P E R F I C I E S.

THE area of any figure, is the measure of its surface, or the space contained within the bounds of the surface, without any regard to thickness.

The area is estimated by the number of squares contained in the surface, the side of those squares being either an inch, or a foot, or yard, &c. And hence the area is said to be so many square inches, or square feet, or square yards, &c.

Our ordinary lineal measures, or measures of length, are as in the first table here below; and the annexed table of square measures is taken from it, by squaring the several numbers.

Lineal Measures.

12 inches	.	1 foot
3 feet	∴	1 yard
6 feet	∴	1 fathom
16½ feet, or	} {	1 pole
5½ yards		or rod
40 poles	.	1 furlong
8 furlongs	.	1 mile

Square Measures.

144 inches	.	1 foot
9 feet	∴	1 yard
36 feet	∴	1 fathom
272¼ feet or	} {	1 pole
30¼ yards		or rod
1600 poles	.	1 furlong
64 furlongs	.	1 mile

P R O-

PROBLEM I.

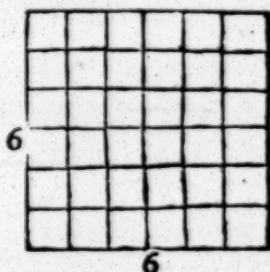
To find the Area of a Parallelogram; whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid.

Multiply the length by the breadth, or perpendicular height, and the product will be the area.

EXAMPLES.

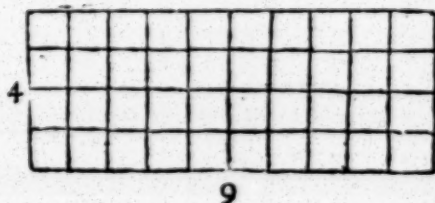
1. To find the area of a square, whose side is 6 inches, or 6 feet, &c.

$$\begin{array}{r} 6 \\ 6 \\ \hline 36 \\ \hline \text{Answer } 36 \end{array}$$



2. To find the area of a rectangle, whose length is 9, and breadth 4 inches, or feet, &c.

$$\begin{array}{r} 9 \\ 4 \\ \hline 36 \\ \hline \text{Answer } 36 \end{array}$$



6

3. To

3. To find the area of a rhombus, whose length is 6.20 chains, and perpendicular height 5.45

$$\begin{array}{r}
 - 5.45 \\
 6.20 \\
 \hline
 10900 \\
 3270 \\
 \hline
 10 \overline{) 33.7900} \\
 3379 \\
 \hline
 4 \\
 \hline
 1.516 \\
 40 \\
 \hline
 20.640 \\
 \hline
 \end{array}$$

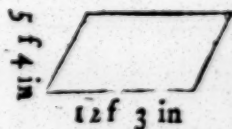


Ans. 3 acres, 1 rood, 20 perches.

Note. Here the square chains are divided by 10 to bring them to acres, because ten square chains make an acre. Also the decimals of an acre are multiplied by 4 roods, and these by 40 perches, because 4 roods make 1 acre, and 40 perches 1 rood.

4. To find the area of the rhomboid, whose length is 12 feet 3 inches, and breadth 5 feet 4 inches.

$$\begin{array}{r}
 f \quad i \\
 12 \quad 3 \\
 5 \quad 4 \\
 \hline
 61 \quad 3 \\
 4 \quad 1 \\
 \hline
 65 \quad 4 \\
 \hline
 \end{array}$$



Answer 65 $\frac{1}{4}$ square feet.

5. To

5. To find the area of a square, whose side is 35·25 chains.

Ans. 124 ac 1 ro 1 perch.

6. To find the area of a parallelogram, whose length is 12·25 chains, and breadth 8·5 chains.

Ans. 10 ac 1 ro 26 perch.

7. To find the area of a rectangular board, whose length is 12·5 feet, and breadth 9 inches.

Ans. $9\frac{3}{8}$ feet

8. To find the square yards of painting in a rhomboid, whose length is 37 feet, and breadth $5\frac{1}{4}$ feet.

Ans. $21\frac{7}{8}$ square yards.

PROBLEM II.

To find the Area of a Triangle.

Rule 1. Multiply the base by the perpendicular height, and take half the product for the area.

Rule 2. When the three sides only are given: Add the three sides all together, and take half the sum; from the half sum subtract each side separately; multiply the half sum and the three remainders continually together; and take the square root of the last product for the area of the triangle.

EXAMPLES.

1. Required the area of the triangle, whose base is 6·25 chains, and perpendicular height 5·20 chains.

6·25

$$\begin{array}{r}
 6.25 \\
 5.20 \\
 \hline
 12500 \\
 3125 \\
 \hline
 20 \overline{) 325000} \\
 \underline{1625} \\
 4 \\
 \hline
 2500 \\
 40 \\
 \hline
 20000
 \end{array}$$

Anf. 1 ac 2 ro 20 perches,

2. To find the number of square yards in the triangle whose three sides are 13, 14, 15 feet.

$$\begin{array}{r}
 13 \\
 14 \\
 15 \\
 \hline
 2 \overline{) 42}
 \end{array}$$



$$\begin{array}{r}
 \frac{1}{2} \text{ sum } 21 \quad 21 \quad 21 \quad 21 \\
 13 \quad 14 \quad 15 \quad 6 \\
 \hline
 \text{remainders } 8 \quad 7 \quad 6 \quad 126 \\
 \hline
 7
 \end{array}$$

$$\begin{array}{r}
 882 \\
 8 \\
 \hline
 9 \overline{) 7056} \quad (84 \text{ feet} \\
 64 \quad 9\frac{1}{3} \text{ sq. yds.}
 \end{array}$$

$$\begin{array}{r}
 164 \mid 656 \\
 4 \mid 656
 \end{array}$$

Anf. $9\frac{1}{3}$ square yards.

3. How

3. How many square yards are in a right-angled triangle, whose base is 40, and perpendicular 30 feet?

Ans. $66\frac{2}{3}$ square yards.

4. To find the area of the triangle, whose three sides are 20, 30, 40 chains.

Ans. 29 ac 0 ro 7 per.

5. How many square yards contains the triangle, whose base is 49 feet, and height $25\frac{1}{2}$ feet?

Ans. $68\frac{1}{2}$ or 68.7361.

6. How many acres, &c. in the triangle, whose three sides are 380, 420, 765 yards?

Ans. 9 ac 0 ro 38 per.

7. To find the area of the triangle, whose base is 18 feet 4 inches, and height 11 feet 10 inches.

Ans. 108 feet 5 inches 8".

8. How many acres, &c. contains the triangle, whose three sides are 49.00, 50.25, 25.69 chains?

Ans. 61 ac 1 ro 39.68 per.



PROBLEM III.

To find one Side of a Right-angled Triangle, having the other two Sides given.

The square of the hypotenuse is equal to both the squares of the two legs. Therefore,

1. To find the hypotenuse; add the squares of the two legs together, and extract the square root of the sum.

2. To find one leg; subtract the square of the other leg from the square of the hypotenuse, and extract the square root of the difference.

EXAMPLES.

1. Required the hypotenuse of a right-angled triangle whose base is 40, and perpendicular 30.

F

40

$$\begin{array}{r}
 0 \quad 30 \\
 40 \quad 30 \\
 \hline
 1600 \quad 900 \\
 900 \\
 \hline
 2500 \text{ (50 the hypotenuse AC)} \\
 25 \\
 \hline
 00
 \end{array}$$



2. What is the perpendicular of a right-angled triangle, whose base AB is 56, and hypotenuse AC 65?

$$\begin{array}{r}
 56 \quad 65 \\
 56 \quad 65 \\
 \hline
 336 \quad 325 \\
 280 \quad 390 \\
 \hline
 3136 \quad 4225 \\
 3136 \\
 \hline
 1089 \text{ (33 the perp. BC.)} \\
 9 \\
 \hline
 63 \mid 189 \\
 3 \mid 189
 \end{array}$$

3. Required the length of a scaling ladder to reach the top of a wall whose height is 28 feet, the breadth of the ditch before it being 45 feet. Ans. 53 feet.

4. To find the length of a shoar, which, strutting 12 feet from the upright of a building, may support a jaumb 20 feet from the ground. Ans. 23'32380 feet.

5. A line of 320 feet will reach from the top of a precipice, standing close by the side of a brook, to the opposite bank : required the breadth of the brook ; the height of the precipice being 103 feet. Ans. 302'9703 feet.

6. A

6. A ladder of 50 feet long being placed in a street, reached a window 28 feet from the ground on one side; and by turning the ladder over, without removing the foot out of its place, it touched a moulding 36 feet high on the other side: required the breadth of the street?

Ans. $76\frac{1}{2}33335$ feet.

PROBLEM IV.

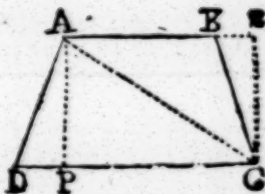
To find the Area of a Trapezoid.

Multiply the sum of the two parallel sides by the perpendicular distance between them, and take half the product for the area.

EXAMPLES.

1. In a trapezoid the parallel sides are AB 7.5, and DC 12.25, also the perpendicular distance AP or Cn is 15.4 chains: required the area.

$$\begin{array}{r}
 12.25 \\
 7.5 \\
 \hline
 19.75 \\
 15.4 \\
 \hline
 7900 \\
 9875 \\
 1975 \\
 \hline
 20 \overline{) 304150} \\
 \underline{152075} \\
 4 \\
 8300 \\
 40 \\
 \hline
 332000
 \end{array}$$



Ans. 15 ac 0 ro 33.2 perches.

2. How many square feet contains the plank, whose length is 12 feet 6 inches, the breadth at the greater end 1 foot 3 inches, and at the less end 11 inches?

Ans. $13\frac{13}{14}$ feet
3. Re-

3. Required the area of a trapezoid, the parallel sides being 21 feet 3 inches and 18 feet 6 inches, and the distance between them 8 feet 5 inches.

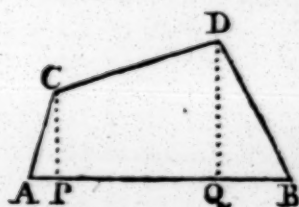
Anf. 167 feet 3 inches, 4" 6".

4. In measuring along one side AB of a quadrangular field, that side and the two perpendiculars upon it from the opposite corners, measured as below: required the content.

Anf. 4 ac 3 r 17' 92 p.

chains

AP	=	1'10
AQ	=	7'45
AB	=	11'10
PC	=	3'52
QD	=	5'95



PROBLEM V.

To find the Area of a Trapezium.

CASE I.

For any Trapezium.

Divide it into two triangles by a diagonal; then find the areas of these triangles, and add them together.

Note. If two perpendiculars be let fall on the diagonal, from the other two opposite angles, the sum of these perpendiculars being multiplied by the diagonal, half the product will be the area of the trapezium.

CASE 2.

When the Trapezium can be inscribed in a Circle.

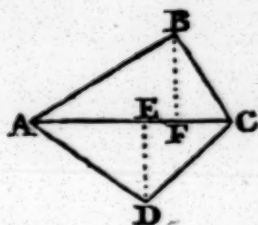
Add all the four sides together, and take half the sum, and subtract each side separately from the half sum; then multiply the four remainders continually together, and the square root of the last product will be the area of the trapezium.

E X.

EXAMPLES.

1. To find the area of the trapezium ABCD, the diagonal AC being 42, the perpendicular BF 18, and the perpendicular DE 16.

$$\begin{array}{r} 18 \\ 16 \\ \hline 34 \\ 42 \\ \hline 68 \\ 136 \\ \hline \end{array}$$



$$2) 1428$$

714 the answer.

2. In the trapezium ABCD, the side AB is 15, BC 13, CD 14, AD 12, and the diagonal AC is 16: required the area.

$$\begin{array}{l} AC \ 16 \\ AB \ 15 \\ BC \ 13 \\ \hline \end{array}$$

$$\begin{array}{l} AC \ 16 \\ CD \ 14 \\ AD \ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \ 44 \\ 22 \ 22 \ 22 \text{ half sum} \\ 16 \ 15 \ 13 \\ \hline 6 \ 7 \ 9 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ 9 \\ \hline 378 \\ 22 \\ \hline \end{array}$$

$$\begin{array}{r} 756 \\ 756 \\ \hline \end{array}$$

$$8316 (91'1921$$

$$\begin{array}{r} 2) \ 42 \\ 21 \ 21 \ 21 \text{ half sum} \\ 16 \ 14 \ 12 \\ \hline 5 \ 7 \ 9 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ 9 \\ \hline 315 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 315 \\ 630 \\ \hline \end{array}$$

$$6615 (81'3326$$

r 3

The

The triangle ABC - - 91'1921

The triangle ADC - - 81'3326

The trapezium ABCD 172'5247 the answer.

3. If a trapezium can be inscribed in a circle, and have its four sides 24, 26, 28, 30; required its area.

24			
26			
28			
30			
2) 108			
54	54	54	54 half sum
24	26	28	30
30	28	26	24
28	24	24	
840	104	52	
	624		
	840		
	24960		
	4992		
	524160		

24

524160 (723'9889488 answer.

4. How many square yards of paving are in the trapezium, whose diagonal is 65 feet, and the two perpendiculars let fall on it 28 and 33'5 feet? Anf. 222 $\frac{1}{2}$ yards.

5. What is the area of a trapezium, whose south side is 27'40 chains, east side 35'75 chains, north side 37'55 chains, west side 41'05 chains, and the diagonal from south-west to north east 48'35 chains?

Anf. 123 ac 0 ro 11'8672 per.

6. What

6. What is the area of a trapezium, whose diagonal is $108\frac{1}{2}$ feet, and the perpendiculars $56\frac{1}{2}$ and $60\frac{3}{4}$ feet?

Ans. $6347\frac{1}{4}$ feet.

7. What is the area of a trapezium inscribed in a circle, the four sides being 12, 13, 14, 15?

Ans. 180.9972372 .

8. In the four-sided field ABCD, on account of obstructions in the two sides AB, CD, and in the perpendiculars BF, DE, the following measures only could be taken, namely, the two sides BC 265 and AD 220 yards, the diagonal AC 378 yards, and the two distances of the perpendiculars from the ends of the diagonal, namely AE 100, and CF 70 yards: required the area in acres, when 4840 square yards make an acre.

Ans. 17 ac 2 ro 21 per.

PROBLEM VI.

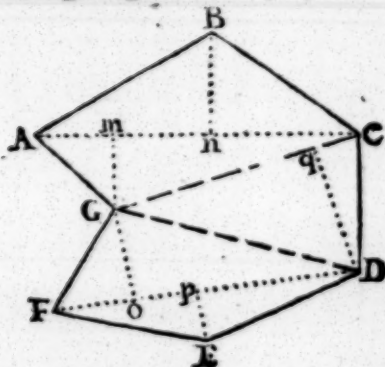
To find the Area of an Irregular Polygon.

Draw diagonals dividing the figure into trapeziums and triangles. Then find the areas of all these separately, and their sum will be the content of the whole irregular figure.

EXAMPLE.

To find the content of the irregular figure ABCDEFGA, in which are given the following diagonals and perpendiculars: namely

AC	5.5
FD	5.2
GC	4.4
Gm	1.3
Bn	1.8
Go	1.2
EP	0.8
DQ	2.3



1st, For trapez. ABCG.	2d, For trapez. GDEF.	3d, For triangle GCD.
1.3	1.2	4.4
1.8	0.8	2.3
<hr/>	<hr/>	<hr/>
3.1	2.0	13.2
5.5	5.2	88
<hr/>	<hr/>	<hr/>
155	10.4	10.12
155	<hr/>	<hr/>
<hr/>		
17.05	double ABCG	
10.40	double GDEF	
10.12	double GCD	
<hr/>		
2) 37.57	double the whole	
18.785	the answer.	
<hr/>		

PROBLEM VII.

To find the Area of a Regular Polygon.

RULE 1.

Multiply the perimeter of the figure, or sum of its sides, by the perpendicular falling from its centre on one of its sides, and half the product will be the area.

RULE 2.

Square the side of the polygon; multiply that square by the multiplier set against its name in the following table, and the product will be the area.

No.

No. of sides.	Names.	Multpliers.
3	Trigon or equ. tri.	0.4330127
4	Tetragon or square	1.0000000
5	Pentagon	1.7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
10	Decagon	7.6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

EXAMPLES.

1. Required the area of the regular pentagon, whose side AB is 25 feet, and perpendicular CP 17.204774.

By the 1st Rule.

17.204774 perp.

125 perim.

86023870
34409548
17204774

2) 2150.596750
1075.298375 anf.

By the 2d Rule.

First 25

25

125

50

625

Then 17.204774

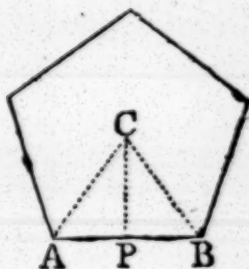
625

86023870
34409548
103228644

1075.2983750 answer.

5

2. To



2. To find the area of the hexagon, whose side is 20.
Anf. 1039'23048.
3. To find the area of the trigon, or equilateral triangle, whose side is 20.
Anf. 173'20508.
4. Required the area of an octagon, whose side is 20.
Anf. 1931'37084.
5. What is the area of a decagon, whose side is 20?
Anf. 3077'68352.

PROBLEM VIII.

To find the Diameter and Circumference of a Circle, the one from the other.

RULE 1.

As 7 is to 22, so is the diameter to the circumference.
As 22 is to 7, so is the circumference to the diameter.

RULE 2.

As 113 is to 355, so is the diameter to the circumf.
As 355 is to 113, so is the circumf. to the diameter.

RULE 3.

As 1 is to 3'1416, so is the diameter to the circumf.
As 3'1416 is to 1, so is the circumf. to the diameter.

EXAMPLES.

1. To find the circumference of a circle, whose diameter AB is 10.

By Rule 1.

$$\begin{array}{r}
 7 : 22 :: 10 : 31\frac{1}{2} \\
 \hline
 7 \overline{) 220} \\
 \underline{31\frac{1}{2}} \\
 \text{or } 31'42857 \text{ anf.}
 \end{array}$$



By

By Rule 2.

$$113 : 355 :: 10 : 31^{\frac{47}{11}}$$

$$113 \overline{) 3550} \quad (31^{\frac{47}{11}} 593$$

160 the ans.

470

180

670

1050

330

By Rule 3.

$$1 : 3^{\circ}14'16'' :: 10 : 31^{\circ}4'16''$$

the circumference nearly,
the true circumference
being
 $31^{\circ}4'159265358979$ &c.

So that the 2d rule is nearest the truth.

2. To find the diameter when the circumference is 50.

By Rule 1.

$$22 : 7 :: 50 : \frac{7 \times 25}{11} = \frac{175}{11} = 15^{\frac{10}{11}} = 15^{\circ}90'0'' \text{ ans.}$$

By Rule 2.

$$355 : 113 :: 50 : 15^{\frac{10}{11}}$$

$$355 \overline{) 5650}$$

74 1130 (15^{\circ}9'155

420

650

110

390

350

By Rule 3.

$$3^{\circ}14'16'' : 1 :: 50 : 15^{\circ}9'156$$

$$3^{\circ}14'16'' \overline{) 50^{\circ}000} \quad (15^{\circ}9'156$$

18484

2876

49

18

2

3. If the diameter of the earth be 7958 miles, as it is very nearly, what is the circumference, supposing it to be exactly round?

Ans. 25000^{\circ}8528 miles.

4. To find the diameter of the globe of the earth, supposing its circumference to be 25000 miles.

Ans. 7957^{\frac{1}{4}} nearly.

PROBLEM IX.

To find the Length of any Arc of a Circle.

RULE 1.

As 180 is to the number of degrees in the arc,

So is 3.1416 times the radius, to its length.

Or as 3 is to the number of degrees in the arc,

So is .05236 times the radius, to its length.

Ex. 1. To find the length of an arc ADB of 30 degrees, the radius being 9 feet.

$$\begin{array}{r}
 3.1416 \\
 \times 9 \\
 \hline
 28.2744 \\
 \text{As } 180 : 30 :: 28.2744 : 4.7124 \\
 \text{Or } 6 : 1 :: 28.2744 : 4.7124 \\
 \text{Or } 3 : 30 :: .05236 \times 9 : 4.7124 \\
 \hline
 4.7124 \text{ the answer}
 \end{array}$$



RULE 2.

From 8 times the chord of half the arc subtract the chord of the whole arc, and $\frac{1}{3}$ of the remainder will be the length of the arc nearly.

Ex. 2. The chord AB of the whole arc being 4.65874, and the chord AD of the half arc 2.34947; required the length of the arc.

$$\begin{array}{r}
 2.34947 \\
 \times 8 \\
 \hline
 18.79576 \\
 - 4.65874 \\
 \hline
 14.13702 \\
 3 \overline{) 14.13702} \\
 \underline{4.71234} \text{ answer.}
 \end{array}$$

Ex. 3. Required the length of an arc of 12 degrees 10 minutes, or $12\frac{1}{3}$ degrees, the radius being 10 feet.

Ans. $2.1234\frac{8}{9}$.

Ex. 4.

Ex. 4. To find the length of an arc whose chord is 6, and the chord of its half is $3\frac{1}{2}$ Ans. $7\frac{1}{4}$.

Ex. 5. Required the length of the arc, whose chord is 8, and the height PD 3. Ans. $10\frac{2}{3}$.

Ex. 6. Required the length of the arc, whose chord is 6, the radius being 9. Ans. 6.11706.

PROBLEM X.

To find the Area of a Circle.

The area of a circle may be found from the diameter and circumference together, or from either of them alone, by these rules following.

Rule 1. Multiply half the circumference by half the diameter. Or multiply the whole circumference by the whole diameter, and take $\frac{1}{4}$ of the product.

Rule 2. Multiply the square of the diameter by $\cdot 7854$.

Rule 3. Multiply the square of the circumference by $\cdot 07958$.

Rule 4. As 14 to 11, so is the square of the diameter to the area.

Rule 5. As 88 to 7, so is the square of the circumference to the area.

EXAMPLES.

1. To find the area of a circle whose diameter is 10, and circumference 31.4159265.

By Rule 1.	By Rule 2	By Rule 4.
$ \begin{array}{r} 31.4159265 \\ \times 10 \\ \hline 314.159265 \\ 4) 314.159265 \\ \text{area } 78.539816 \end{array} $	$ \begin{array}{r} .7854 \\ \times 100 \\ \hline 78.54 \end{array} $	$ \begin{array}{r} 100 \\ 11 \\ \hline 14 \overline{) 1100} \\ 7 \overline{) 550} \\ \hline 78.57 \end{array} $
		By

*By Rule 3.*sq. circ. 986·96044
invert. 85970

6908723
888264
49348
7896

78·54231 area
*By Rule 5*31·4159265 circum.
562951413 invert.

94247779
3141593
1256637
31416
15708
2827
63
19
2

88 : 7 :: 986·96044
7

8 | 69·8·72308
11 | 863·59038
78 50821

Ex. 2. Required the area of the circle, whose diameter is 7, and circumference 22. Ans. $38\frac{1}{2}$.

Ex. 3. What is the area of a circle, whose diameter is 1, and circumference 3·1416? Ans. ·7854.

Ex. 4. What is the area of a circle, whose diameter is 7? Ans. $38\cdot4846$.

Ex. 5. How many square yards are in a circle whose diameter is $3\frac{1}{2}$ feet? Ans. 1·069.

Ex. 6. How many square feet does a circle contain, the circumference being 10·9956 yards? Ans. $86\cdot19266$.

PROBLEM XI.

To find the Area of the Sector of a Circle.

RULE I.

Multiply the radius, or half the diameter, by half the arc of the sector, for the area. Or, multiply the diameter by the arc of the sector, and take $\frac{1}{4}$ of the product.

Note. The arc may be found by problem ix.

RULE

RULE 2.

As 360 is to the degrees in the arc of the sector, so is the whole area of the circle, to the area of the sector.

Note. For a semicircle take one half, for a quadrant one quarter, &c. of the whole circle.

EXAMPLES.

1. What is the area of the sector CADB, the radius being 10, and the chord AB 16?

$$100 = AC^2$$

$$64 = AE^2$$

$$36 \{ 6 = CE$$

$$10 = CD$$

$$4 = DE$$

$$16 = DE^2$$

$$64 = AE^2$$

$$80 \{ 8.9442719 = AD$$

$$71.5541752$$

$$16$$

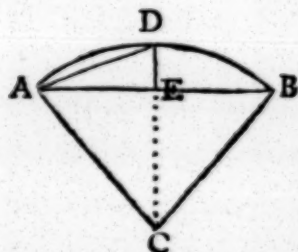
$$3 \} 55.5541752$$

$$2 \} 18.5180584 \text{ arc ADB}$$

$$9.2590297 = \text{half arc}$$

$$10 = \text{radius}$$

$$92.590297 \text{ answer.}$$



Ex. 2.

Ex. 2. Required the area of the sector, whose arc contains 18 degrees; the diameter being 3 feet.

$$\begin{array}{r} .7854 \\ 9 \\ \hline \end{array}$$

Then, as $360 : 18 :: 7.0686$ the area of the whole circle.
Or as $20 : 1 :: 7.0686 : .35343$ the answer.

Ex. 3. What is the area of the sector, whose radius is 10, and arc 20? Ans. 100.

Ex. 4. What is the area of the sector, whose radius is 9, and the chord of its arc 6? Ans. 27.52678.

Ex. 5. Required the area of a sector, whose radius is 25, its arc containing 147 degrees 29 minutes.

Ans. 804.4017.

Ex. 6. To find the area of a quadrant and a semicircle, to the radius 13. Ans. 132.7326 and 265.4652.

PROBLEM XII.

To find the Area of a Segment of a Circle.

RULE I.

Find the area of the sector having the same arc with the segment, by the last problem.

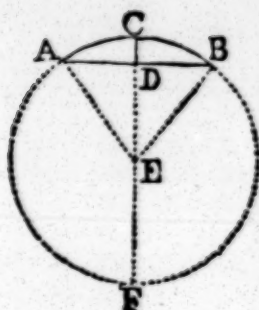
Find also the area of the triangle, formed by the chord of the segment and the two radii of the sector.

Then the sum of these two will be the answer when the segment is greater than a semicircle; but the difference will be the answer when it is less than a semicircle.

EXAMPLE I.

Required the area of the segment ACBDA, its chord AB being 12, and the radius AE or CE 10.

$$\begin{array}{r}
 100 \text{ AE}^2 \\
 36 \text{ AD}^2 \\
 \hline
 64 \text{ DE}^2 \\
 \hline
 \text{its root } 8 \text{ DE} \\
 \text{from } 10 \text{ CE} \\
 \hline
 2 \text{ CD} \\
 \hline
 4 \text{ CD}^2 \\
 36 \text{ AD}^2 \\
 \hline
 40 \text{ chord AC}^2 \\
 \hline
 \text{its root } 6.324555 \text{ chord AC} \\
 8 \\
 \hline
 50.596440 \\
 12 \cdot \\
 \hline
 3) 38.39644 \\
 2) 12.86548 \text{ arc ACB} \\
 6.43274 \text{ half arc} \\
 10 \text{ radius} \\
 \hline
 64.3274 \text{ area of sect. or EACB} \\
 48.0000 \text{ area of triangle EAB} \\
 \hline
 \text{anf. } 16.3274 \text{ area of segm. ACBA}
 \end{array}$$



$$\begin{array}{r}
 6 \text{ AD} \\
 8 \text{ DE} \\
 \hline
 48 \text{ area of } \triangle \text{ EAB}
 \end{array}$$

R U L E 2.

To the chord of the whole arc add $\frac{1}{3}$ of the chord of half the arc, or add the latter chord and $\frac{1}{3}$ of it more.

Multiply the sum by the versed sine or height of the segment, and take $\frac{2}{3}$ of the product for the area of the segment.

Ex.

Ex. Take the same example, in which the radius is 10, and the chord AB 12.

Then, as before, are found CD 2, and the chord of the half arc AC

Hence $\frac{1}{4}$ is 2.108185

AB 12.

$$\begin{array}{r} 20.432740 \\ \text{CD} \quad - \quad - \quad 2 \end{array}$$

$$\begin{array}{r} 40.86548 \\ \quad \quad \quad .4 \end{array}$$

Anf. 16.346192 area nearly.

RULE 3.

Divide the height of the segment by the diameter, and find the quotient in the column of heights or versed sines, at the end of the book.

Take out the corresponding area in the next column on the right hand, and multiply it by the square of the diameter, for the answer.

Ex. The example being the same as before, we have CD equal to 2, and the diameter 20.

Then 20) 2 (.1

And 10 .1 answers .040875

Sq. of diam. - - - 400

Answer 16.3500

OTHER EXAMPLES.

Ex. 2. What is the area of the segment, whose height is 2, and the chord 20? Anf. 26.878787.

Ex. 3. What is the area of the segment, whose height is 18, and diameter of the circle 50? Anf. 636.375.

Ex. 4. Required the area of the segment whose chord is 16, the diameter being 20. Anf. 44.7292.

P R O.

PROBLEM XIII.

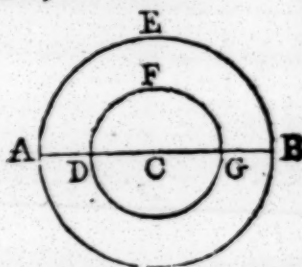
To find the Area of a Circular Ring, or Space included between two Concentric Circles.

Take the difference between the two circles, for the ring; or multiply the sum of the diameters by their difference, and multiply the product by .7854, for the answer.

EXAMPLES.

1. The diameters of the two concentric circles being AB 10 and DG 6, required the area of the ring contained between their circumferences AEBA, and DFGD.

	10		.7854
	6		64
	—		—
sum	16		31416
dif.	4		47124
	—		—
	64		50.2656 anf.
	—		—



Ex. 2. The diameters of two concentric circles being 20 and 10; required the area of the ring between their circumferences.

Anf. 235.62.

Ex. 3. What is the area of a ring, the diameters of whose bounding circles are 6 and 4?

Anf. 15.708.

PROBLEM XIV.

To measure long Irregular Figures.

Take the breadth in several places, at equal distances. Add all the breadths together, and divide the sum by the number of them, for the mean breadth; which multiply by the length for the area.

EXAM-

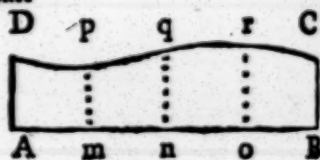
But Note. That in this rule half the sum of the first and last breadths is taken but for one only.

If the breadths be not taken at equal distances; then compute all the little trapezoids separately, and add them all together.—Or, add all the breadths together, and divide the sum by the whole number of them for the mean breadth, to multiply by the length for the whole area, which will not be far from the truth.

EXAMPLES.

1. The breadths of an irregular figure, at five equidistant places being AD 8.2, mp 7.4, nq 9.2, or 10.2 BC 8.6; and the length AB 39; required the area.

$$\begin{array}{r}
 8.2 \\
 8.6 \\
 \hline
 2 \) \ 16.8 \\
 \hline
 8.4 \text{ mean of first and last} \\
 7.4 \\
 9.2 \\
 10.2 \\
 \hline
 4 \) \ 35.2 \\
 8.8 \text{ mean of all} \\
 39 \\
 \hline
 343.2 \text{ answer.}
 \end{array}$$



Ex. 2. The length of an irregular figure being 84, and the breadths at 6 equidistant places 17.4, 20.6, 14.2, 16.5, 20.1, 24.4? what is the area? Ans. 1550.64.

MENSURATION OF SOLIDS.

DEFINITIONS.

SOLIDS, or bodies, are figures having length, breadth, and thickness,

2. A prism is a solid, or body, whose ends are any plane figures, which are parallel, equal, and similar; and its sides are parallelograms.

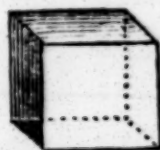
A prism is called a triangular one, when its ends are triangles; a square prism, when its ends are squares; a pentagonal prism, when its ends are pentagons; and so on.

3. A cube is a square prism, having six sides, which are all squares. It is like a die, having its sides perpendicular to one another.

4. A parallelopipedon is a solid having six rectangular sides, every opposite pair of which are equal and parallel.

5. A cylinder is a round prism, having circles for its ends.

6. A pyramid is a solid having any plane figure for a base, and its sides are triangles whose vertices meet in a point at the top, called the vertex of the pyramid.



The

The pyramid takes names according to the figure of its base, like the prism; being triangular, or square, or hexagonal, &c.

7. A cone is a round pyramid, having a circular base.



8. A sphere is a solid bounded by one continued convex surface, every point of which is equally distant from a point within, called the centre.—The sphere may be conceived to be formed by the revolution of a semi-circle about its diameter, which remains fixed.

9. The axis of a solid, is a line drawn from the middle of one end, to the middle of the opposite end; as between the opposite ends of a prism. Hence the axis of a pyramid, is the line from the vertex to the middle of the base, or the end on which it is supposed to stand. And the axis of a sphere, is the same as a diameter, or a line passing through the centre, and terminated by the surface on both sides.

10. When the axis is perpendicular to the base, it is a right prism or pyramid; otherwise, it is oblique.

11. The height or altitude of a solid, is a line drawn from its vertex or top, perpendicular to its base.—This is equal to the axis in a right prism or pyramid; but in an oblique one, the height is the perpendicular side of a right-angled triangle, whose hypotenuse is the axis.

12. Also a prism or pyramid is regular or irregular, as its base is a regular or an irregular plane figure.

13. The segment of a pyramid, sphere, or any other solid, is a part cut off the top by a plane parallel to the base of that figure.

14. A frustum or trunk, is the part that remains at the bottom, after the segment is cut off.

15. A zone of a sphere, is a part intercepted between two parallel planes; and is the difference between two segments. When the ends, or planes, are equally distant from the centre, on both sides, the figure is called the middle zone.

16. The sector of a sphere, is composed of a segment less than a hemisphere or half sphere, and of a cone having the same base with the segment, and its vertex in the centre of the sphere.

17. A circular spindle, is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed.



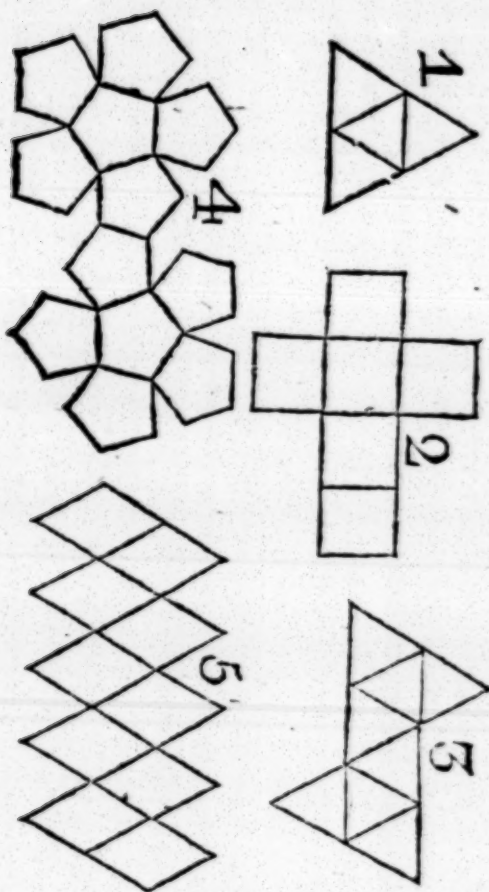
18. A regular body, is a solid contained under a certain number of equal and regular plane figures of the same sort.

19. The faces of the solid are the plane figures under which it is contained. And the linear sides, or edges of the solid, are the sides of the plane faces.

20. There are only five regular bodies: namely, 1st, the tetraedron, which is a regular pyramid, having four triangular faces: 2d, the hexaedron, or cube, which has 6 equal square faces; 3d, the octaedron, which has 8 triangular faces: 4th, the dodecaedron, which has 12 pentagonal faces; 5th, the icosaedron, which has 20 triangular faces.

Note, If the following figures be exactly drawn on pasteboard, and the lines cut half through, so that the parts be turned up and glued together, they will represent the five regular bodies: namely, figure 1 the tetraedron, figure 2 the hexaedron, figure 3 the octaedron, figure 4 the dodecaedron, and figure 5 the icosaedron.

Fig. 1.



Note also, that, in cubic measure,
 1728 inches make 1 foot
 27 feet - - 1 yard
 166 $\frac{2}{3}$ yards - - 1 pole
 64000 poles - - 1 furlong
 512 furlongs - 1 mile.

PROBLEM I.

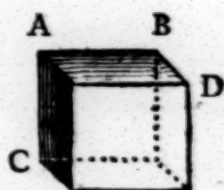
To find the Solidity of a Cube.

Cube one of its sides for the content; that is, multiply the side by itself, and that product by the side again.

EXAMPLES.

1. If the side AB, or AC, or BD, of a cube be 24 inches, what is its solidity or content?

$$\begin{array}{r}
 24 \\
 24 \\
 \hline
 96 \\
 48 \\
 \hline
 576 \\
 24 \\
 \hline
 2304 \\
 1152 \\
 \hline
 13824 \text{ anf.}
 \end{array}$$



Ex. 2. How many solid feet are in the cube whose side is 22 feet? Anf. 10648.

Ex. 3. Required how many solid feet are in the cube whose side is 18 inches? Anf. $3\frac{3}{8}$.

PROBLEM II.

To find the Solidity of a Parallelopipedon.

Multiply the length, breadth, and depth, or altitude, all continually together, for the solid content; that is, multiply the length by the breadth, and that product by the depth.

G

EX.

EXAMPLES.

1. Required the content of the parallelopipedon, whose length AB is 6 feet, its breadth AC $2\frac{1}{2}$ feet, and altitude BD $1\frac{1}{4}$ feet?

$$\begin{array}{r}
 1.75 \text{ BD} \\
 6 \text{ AB} \\
 \hline
 10.50 \\
 2.5 \text{ AC} \\
 \hline
 52.50 \\
 2100 \\
 \hline
 26.250 \text{ anf.}
 \end{array}$$



Ex. 2. Required the content of a parallelopipedon, whose length is 10.5, breadth 4.2, and height 3.4?

Anf. 149.94.

Ex. 3. How many cubic feet are in a block of marble, whose length is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches?

Anf. $21\frac{1}{3}$.

PROBLEM III.

To find the Solidity of any Prism.

Multiply the area of the base, or end, by the height, or length, and it will give the content.

Which rule will do, whether the prism be triangular, or square, or pentagonal, &c. or round, as a cylinder.

EXAMPLES.

1. What is the content of a triangular prism, whose length AC is 12 feet, and each side AB of its equilateral base $2\frac{1}{2}$ feet?

$$\text{Here } \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}.$$

Then

Then $\cdot 433013$ tabular n°

$6\frac{1}{2}$

$2\cdot 598078$

108253

$2\cdot 706331$ area of end
12 length

$32\cdot 475972$ answer.



Ex. 2. Required the solidity of a triangular prism, whose length is 10 feet, and the three sides of its triangular end or base, are 5, 4, 3 feet? Ans. 60.

Ex. 3. What is the content of a hexagonal prism, the length being 8 feet, and each side of its end 1 foot 6 inches? Ans. $46\cdot 765368$.

Ex. 4. Required the content of a cylinder, whose length is 20 feet, and circumference $5\frac{1}{2}$ feet? Ans. $48\cdot 1459$.

Ex. 5. What is the content of a round pillar, whose height is 16 feet, and diameter 2 feet 3 inches? Ans. $63\cdot 6174$.

PROBLEM IV.

To find the Convex Surface of a Cylinder.

Multiply the circumference by the height or length of the cylinder.

Note. The upright surface of any prism is found in the same manner, viz. by multiplying the perimeter of the end by the length. And the solidity of a cylinder is found as the prism in the last problem.

EXAMPLES.

1. What is the convex surface of a cylinder, whose length is 16 feet, and its diameter 2 feet 3 inches

G 2

$3\cdot 1416$

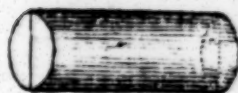
$$\begin{array}{r} 3'1416 \\ 2\frac{1}{4} \text{ diameter} \\ \hline \end{array}$$

$$\begin{array}{r} 6'2832 \\ '7854 \\ \hline \end{array}$$

$$\begin{array}{r} 7'0686 \text{ circumf.} \\ 16 \\ \hline \end{array}$$

$$\begin{array}{r} 424116 \\ 70686 \\ \hline \end{array}$$

113'0976 answer.



Ex. 2. Required the convex surface of the cylinder, whose length is 20 feet, and its diameter 2 feet?

Ans. 125'664.

Ex. 3. What is the convex surface of a cylinder, whose length is 18 feet 6 inches, and circumference 5 feet 4 inches?

Ans. 98 $\frac{2}{3}$.

PROBLEM V.

To find the Convex Surface of a Right Cone.

-- Multiply the circumference of the base by the slant height, or length of the side, and take half the product for the surface.

EXAMPLES.

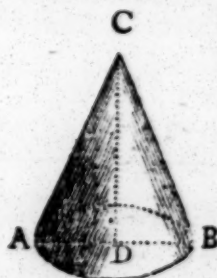
1. If the diameter of the base be AB 5 feet, and the side of the cone AC 18, required the convex surface?

$$\begin{array}{r} 3'1416 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 15'7080 \text{ circumf.} \\ 18 \\ \hline \end{array}$$

$$\begin{array}{r} 125664 \\ 15708 \\ \hline \end{array}$$

$$\begin{array}{r} 2) 282'744 \\ 141'372 \text{ answer.} \\ \hline \end{array}$$



Ex. 2.

Ex. 2. What is the convex surface of a cone, whose side is 20, and the circumference of its base 9?

Anf. 90.

Ex. 3. Required the convex surface of a cone, whose slant height is 50 feet, and the diameter of its base 8 feet 6 inches?

Anf. 667.59.

PROBLEM VI.

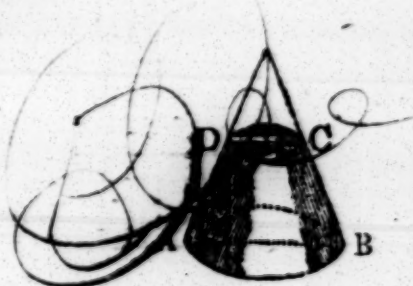
To find the Convex Surface of the Frustum of a Right Cone.

Add together the perimeters of the two ends; then multiply that sum by the slant height, or side of the frustum, and take half the product for the surface.

EXAMPLES.

1. If the circumferences of the two ends be 12.5 and 10.3, and the slant height AD 14, required the convex surface of the frustum ABCD?

$$\begin{array}{r}
 12.5 \\
 10.3 \\
 \hline
 22.8 \text{ sum} \\
 14 \\
 \hline
 912 \\
 228 \\
 \hline
 2) 319.2 \\
 159.6 \text{ answer.}
 \end{array}$$



Ex. 2. What is the convex surface of the frustum of a cone, the slant height of the frustum being 12.5, and the circumferences of the two ends 6 and 8.4? Anf. 90.

Ex. 3. Required the convex surface of the frustum of a cone, the side of the frustum being 10 feet 6 inches, and the circumferences of the two ends 2 feet 3 inches, and 5 feet 4 inches?

Anf. 391.2.

PROBLEM VII.

To find the Solidity of a Cone, or any Pyramid.

Compute the area of the base, then multiply that area by the height, and take $\frac{1}{3}$ of the product for the content.

EXAMPLES.

1. What is the solidity of a cone, whose height CD is $12\frac{1}{2}$ feet, and the diameter AB of the base $2\frac{1}{2}$?

$$\text{Here } 2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}.$$

$$\text{Then } \begin{array}{r} .7854 \\ 6\frac{1}{4} \end{array}$$

$$\begin{array}{r} 4.7124 \\ 19635 \end{array}$$

$$\begin{array}{r} 4.90875 \text{ area of base} \\ 12\frac{1}{2} \text{ height} \end{array}$$

$$\begin{array}{r} 5890500 \\ 2454375 \end{array}$$

$$\begin{array}{r} 3 \text{) } 61.359375 \\ 20.453125 \text{ answer.} \end{array}$$

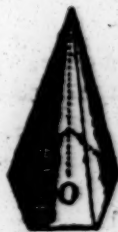


Ex. 2. What is the solid content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?

$$\begin{array}{r} 1.720477 \text{ tab. area} \\ 4 \text{ sq. side} \end{array}$$

$$\begin{array}{r} 6.881908 \text{ area base} \\ 4 \frac{1}{3} \text{ of height} \end{array}$$

$$\begin{array}{r} 27.527632 \text{ answer.} \end{array}$$



Ex. 3.

Ex. 3. What is the content of a cone, its height being $10\frac{1}{2}$ feet, and the circumference of its base 9 feet?

Ans. 22.56093.

Ex. 4. Required the content of a triangular pyramid, its height being 14 feet 6 inches, and the three sides of its base 5, 6, 7?

Ans. 71.0352.

Ex. 5. What is the content of a hexagonal pyramid, whose height is 6.4, and each side of its base 6 inches?

Ans. 1.38564 feet.

PROBLEM VIII.

To find the Solidity of the Frustum of a Cone or any Pyramid.

RULES.

1. Add into one sum, the areas of the two ends, and the mean proportional between them, or the square root of their product; and take $\frac{1}{3}$ of that sum for a mean area; which multiplied by the height of the frustum, will give the content.

2. When the ends are regular plane figures; the mean area will be found by multiplying $\frac{1}{3}$ of the corresponding tabular number belonging to the polygon, either by the sum arising by adding together the square of a side of each end and the product of the two sides, or by the quotient of the difference of their cubes divided by their difference, or by the sum arising from the square of their half difference added to 3 times the square of their half sum.

3. And in the frustum of a cone, the mean area is found by multiplying .2618, or $\frac{1}{3}$ of .7854, either by the sum arising by adding together the squares of the two diameters and the product of the two, or by the difference of their cubes divided by their difference, or by the square of half their difference added to 3 times the square of their half sum.

Or, if the circumferences be used in like manner, instead of their diameters, the multiplier will be $\cdot 02654$, instead of $\cdot 2618$.

EXAMPLES.

1. What is the content of a frustum of a cone, whose height is 20 inches, and the diameters of its two ends 28 and 20 inches?

28	28	20
28	20	20
224	560	400
56	784	
784	400	
	1744	
	$\cdot 2618$	
	13952	
	1744	
	10464	
	3488	
	4565792	
	20	
	1315840	

answer.

*See Fig.
to Prob. vi.*

Ex. 2. Required the content of a pentagonal frustum, whose height is 5 feet, each side of the base 1 foot 6 inches, and each side of the less end 6 inches?

18	18	6
18	6	6
<hr/>	<hr/>	<hr/>
144	108	36
18	324	<hr/>
<hr/>	36	
324	<hr/>	
	3) 468	

156 $\frac{1}{3}$ of sum
1720477 tab. area

10322862
8602385
1720477

268394412 mean area
5 height

144 { 12 | 1341972060
12 | 111831005
9319250 answer in cubic feet.



Ex. 3. What is the solidity of the frustum of a cone, the altitude being 25, the circumference at the greater end 20, and at the less end 10? Ans. 464.205.

Ex. 4. How many solid feet are in a piece of timber, whose bases are squares, each side of the greater end being 15 inches, and each side of the less end 6 inches; also the length, or perpendicular altitude is 24 feet?

Ans. 19 $\frac{1}{2}$.

Ex. 5. To find the content of the frustum of a cone, the altitude being 18, the greatest diameter 8, and the least 4.

Ans. 527.7888.

Ex. 6. What is the solidity of a hexagonal frustum, the height being 6 feet, the side of the greater end 18 inches, and of the less 12 inches?

Ans. 24681722.

PROBLEM IX.

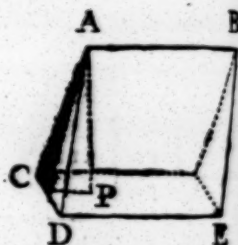
To find the Solidity of a Wedge.

To the length of the edge add twice the length of the back or base, and reserve the sum; multiply the height of the wedge by the breadth of the base; then multiply this product by the reserved sum, and take $\frac{1}{6}$ of the last product for the content.

EXAMPLES.

1. What is the content in feet of a wedge, whose altitude AP is 14 inches, its edge AB 21 inches, and the length of its base DE 32 inches, and its breadth CD $4\frac{1}{2}$ inches?

$$\begin{array}{r}
 21 \\
 32 \\
 32 \\
 \hline
 85 \\
 \hline
 21 \\
 14 \\
 4\frac{1}{2} \\
 \hline
 56 \\
 7 \\
 \hline
 63 \\
 85 \\
 \hline
 315 \\
 504 \\
 \hline
 \end{array}$$



$$\begin{array}{r}
 1728 \left\{ \begin{array}{l} 6 \\ 12 \\ 12 \\ 12 \end{array} \right| \begin{array}{l} 5355 \\ 8925 \text{ anf. in cubic inches} \\ 74375 \\ 6197916 \\ 516493 \text{ anf. in feet, or little more} \\ \hline \end{array}
 \end{array}$$

than half a cubic foot.

Ex. 2. Required the content of a wedge, the length and breadth of the base being 70 and 30 inches, the length of the edge 110 inches, and the height $34\cdot29016$?
Anf. $24\cdot8048$.

PROBLEM X.

To find the Solidity of a Prismoid.

Definition.

A prismoid differs only from the frustum of a pyramid, in not having its opposite ends similar planes.

RULE.

R U L E.

Add into one sum, the areas of the two ends and 4 times the middle section parallel to them, and $\frac{1}{3}$ of that sum will be a mean area; which being multiplied by the height, will give the content.

Note. For the length of the middle section, take half the sum of the lengths of the two ends; and for its breadth, take half the sum of the breadths of the two ends.

E X A M P L E S.

1. How many cubic feet are there in a stone, whose ends are rectangles, the length and breadth of the one being 14 and 12 inches; and the corresponding sides of the other 6 and 4 inches; the perpendicular height being $30\frac{1}{2}$ feet?

14	10	6
12	8	4
—	—	—
168	80	24
—	4	—
	320	
	168	
	24	

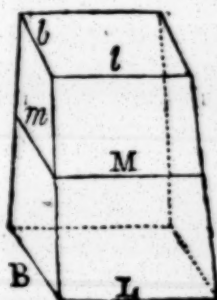
$$6 \overline{) 512}$$

$85\frac{1}{3}$ mean area in inches
 $30\frac{1}{2}$ height

$$\begin{array}{r} 2560 \\ 42\frac{2}{3} \\ \hline \end{array}$$

$$144 \left\{ \begin{array}{l} 12 \\ 12 \end{array} \right| \begin{array}{l} 2602.6 \\ 216.8 \\ \hline 18.074 \text{ answer} \end{array}$$

G 6



Ex. 2.

Ex. 2. Required the content of a rectangular prismoid, whose greater end measures 12 inches by 8, the lesser end 8 inches by 6, and the perpendicular height 5 feet? Ans. 2'453 feet.

Ex. 3. What is the content of a cart or waggon, whose inside dimensions are as follow: at the top the length and breadth $81\frac{1}{2}$ and 55 inches, at the bottom the length and breadth 41 and $29\frac{1}{2}$ inches, and the height $47\frac{1}{4}$ inches? Ans. 126340'59375 cubic inches.

PROBLEM XI.

To find the Convex Surface of a Sphere or Globe.

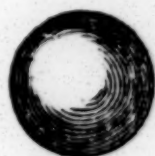
Multiply its circumference by its diameter.

Note. In like manner the convex surface of any zone or segment is found, by multiplying its height by the whole circumference of the sphere.

EXAMPLES.

1. Required the convex superficies of a globe, whose diameter or axis is 24 inches?

$$\begin{array}{r}
 3\cdot1416 \\
 24 \text{ diam.} \\
 \hline
 125664 \\
 62832 \\
 \hline
 75\cdot3984 \text{ circumf.} \\
 24 \\
 \hline
 3015936 \\
 1507968 \\
 \hline
 1809\cdot5616 \text{ answer,}
 \end{array}$$



Ex. 2. What is the convex surface of a sphere, whose diameter is 7, and circumference 22? Ans. 154.

Ex. 3.

Ex. 3. Required the area of the surface of the earth, its diameter, or axis, being $7957\frac{1}{4}$ miles, and its circumference 25000 miles? *Ans.* 198943750 sq. miles.

Ex. 4. The axis of a sphere being 42 inches, what is the convex superficies of the segment, whose height is 9 inches? *Ans.* 1187.5248 inches.

Ex. 5. Required the convex surface of a spherical zone, whose breadth or height is 2 feet, and cut from a sphere of $12\frac{1}{2}$ feet diameter? *Ans.* 78.54 feet.

PROBLEM XII.

To find the Solidity of a Sphere or Globe.

Find the cube of the axis, and multiply it by .5236.

EXAMPLES.

1. What is the solidity of the sphere, whose axis is 12?

	Or thus
12	.5236
12	12
<hr/>	<hr/>
144	6.2832
12	12
<hr/>	<hr/>
1728	75.3984
.5236	12
<hr/>	<hr/>
10368	904.7808 <i>anf.</i>
5184	
3456	
8640	
<hr/>	
904.7808 <i>anf.</i>	

Ex. 2. To find the content of the sphere, whose axis is 2 feet 8 inches. *Ans.* 9.9288 feet.

Ex. 3.

Ex. 3. Required the solid content of the earth, supposing its circumference to be 25000 miles?

Ans. 263858149120 miles.

PROBLEM XIII.

To find the Solidity of a Spherical Segment.

To 3 times the square of the radius of its base, add the square of its height; then multiply the sum by the height, and the product again by .5236.

EXAMPLES.

1. Required the content of a spherical segment, its height being 4 inches, and the radius of its base 8?

8	4	.5236
8	4	832
<hr/>		<hr/>
64	16	10472
3	192	15708
<hr/>		<hr/>
192	208	41888
<hr/>		<hr/>
	4	4356352 ans.
<hr/>		<hr/>
	832	



Ex. 2. What is the solidity of the segment of a sphere, whose height is 9, and the diameter of its base 20?

Ans. 17954244.

Ex. 3. Required the content of the spherical segment, whose height is $2\frac{1}{4}$, and the diameter of its base 8.61684?

Ans. 715695.

PROBLEM XIV.

To find the Solidity of a Spherical Zone or Frustum.

Add together the square of the radius of each end, and $\frac{1}{3}$ of the square of their distance, or of the height; then multiply the sum by the said height, and the product again by 1.5708.

EX.

EXAMPLES.

1. What is the solid content of a zone, whose greater diameter is 12 inches, the less 8, and the height 10 inches?

$$\begin{array}{r}
 6 \qquad 4 \qquad 10 \\
 6 \qquad 4 \qquad 10 \\
 \hline
 36 \qquad 16 \qquad 3 \mid 100 \\
 \hline
 \qquad 36 \qquad 33\frac{1}{3} \\
 \hline
 \qquad 33\frac{1}{3} \\
 \hline
 \qquad 85\frac{1}{3} \\
 1'5708 \\
 \hline
 78540 \\
 125664 \\
 5236 \\
 \hline
 134'0416 \\
 10 \\
 \hline
 1340'416 \text{ anf.}
 \end{array}$$



Ex. 2. Required the content of a zone, whose greater diameter is 12, less diameter 10, and height 2?

Anf. 195'8264.

Ex. 3. What is the content of a middle zone, whose height is 8 feet, and the diameter of each end 6?

Anf. 494'2784 feet.

PROBLEM XV.

To find the Surface of a Circular Spindle.

Multiply the length AB of the spindle by the radius OC of the revolving arc. Multiply also the said arc ACB by the central distance OE, or distance between the

the centre of the spindle and centre of the revolving arc. Subtract the latter product from the former, and multiply double the remainder by 3.1416, or the single remainder by 6.2832, for the surface.

Note. The same rule will serve for any segment or zone cut off perpendicular to the chord of the revolving arc, only using the particular length of the part, and the part of the arc which describes it, instead of the whole length and whole arc.

EXAMPLES.

1. Required the surface of a circular spindle, whose length AB is 40, and its thickness CD 30 inches?

Here, by the notes at pa. 91,

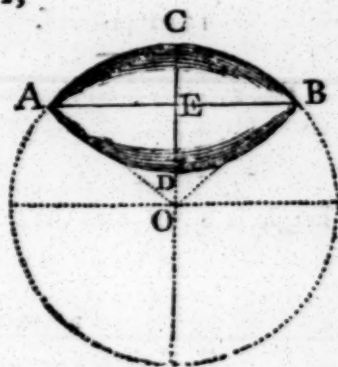
The chord AC = $\sqrt{AE^2 + CE^2} = \sqrt{20^2 + 15^2} = 25$,

and $2CE : AC :: AC : CO = \frac{25^2}{30} = 20\frac{5}{6}$,

hence $OE = OC - CE = 20\frac{5}{6} - 15 = 5\frac{5}{6}$.

Also, by problem x, rule 2,

$$\begin{array}{r}
 25 \text{ AC} \\
 8 \\
 \hline
 200 \\
 40 \text{ AB} \\
 \hline
 3 \overline{) 160} \\
 53\frac{1}{3} \text{ arc ACB} \\
 \hline
 \end{array}$$



Then,

Then, by the rule,

$$\begin{array}{r}
 20\frac{1}{2} \quad 53\frac{1}{3} \\
 40 \quad 5\frac{5}{8} \\
 \hline
 800 \quad 266\frac{2}{3} \\
 33\frac{1}{3} \quad 44\frac{4}{9} \\
 \hline
 833\frac{1}{3} \quad 311\frac{1}{9} \\
 311\frac{1}{9} \\
 \hline
 \quad \quad \quad 4700 \\
 522\frac{2}{9} \text{ or } 522.2 \text{ or } \hline
 6.2832 \quad 9 \\
 \hline
 \quad \quad \quad \text{Or thus,} \\
 10444 \quad 6.2832 \\
 15666 \quad 4700 \\
 \hline
 417777 \quad 439824 \\
 10444444 \quad 251328 \\
 \hline
 31333333 \quad \hline
 9 \quad 29531.04 \\
 3281.2266 \quad 3281.226 \text{ anf. nearly}
 \end{array}$$

Ex. 2. What is the surface of a circular spindle, whose length is 24, and thickness in the middle 18?

Anf. 1177.4485.

PROBLEM XVI.

To find the Solidity of a Circular Spindle.

Multiply the central distance OE by half the area of the revolving segment ACBEA. Subtract the product from $\frac{1}{3}$ of the cube of AE, half the length of the spindle. Then multiply the remainder by 12.5664, or 4 times 3.1416, for the whole content.

EXAMPLES.

1. Required the content of the circular spindle, whose length AB is 40, and middle diameter CD 30?

[See the last figure.]

By

By the work of the last problem,

we have OE = $6\frac{2}{3}$	20 half length
and arc AC = $26\frac{2}{3}$	20
and rad. OC = $20\frac{2}{3}$	
<hr/>	<hr/>
533 $\frac{1}{3}$	400
22 $\frac{2}{3}$	20
<hr/>	<hr/>
Sector OACB 555 $\frac{1}{3}$	3) 8000
AE \times OE = OAB 116 $\frac{2}{3}$	<hr/>
<hr/>	2666 $\frac{2}{3}$
2) 438 $\frac{8}{9}$	1280 $\frac{2}{3}$
<hr/>	<hr/>
$\frac{1}{2}$ seg. ACE 219 $\frac{4}{9}$	1386 $\frac{4}{9}$
OE 5 $\frac{2}{3}$	<hr/>
<hr/>	or 1386.44
1097 $\frac{2}{9}$	4665.21 mult. inver.
183 nearly	<hr/>
<hr/>	138644
1280 $\frac{2}{3}$	27739
<hr/>	6932
	832
	83
	5
	<hr/>
	17423.5 ans.

Ex. 2. What is the solidity of a circular spindle, whose length is 24, and middle diameter 18?

Ans. 3739.93.

PROBLEM XVII.

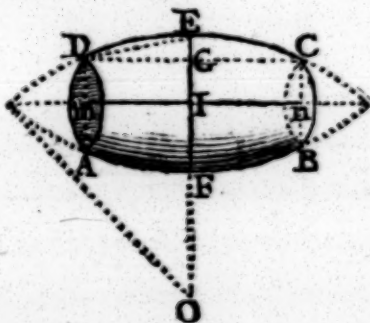
To find the Solidity of the Middle Frustum or Zone of a Circular Spindle.

From the square of half the length of the whole spindle, take $\frac{1}{3}$ of the square of half the length of the middle frustum, and multiply the remainder by the said half length of the frustum.—Multiply the central distance by

by the revolving area, which generates the middle frustum.
—Subtract this latter product from the former; then the remainder multiplied by 6.2832, or 2 times 3.1416, will give the content.

EXAMPLES.

1. Required the solidity of the frustum, whose length mn is 40 inches, also its greatest diameter EF is 32, and least diameter AD or BC 24?



Draw DG parallel to mn, then we

have $DG = \frac{1}{2}mn = 20$,

and $EG = \frac{1}{2}EF - \frac{1}{2}AD = 4$,

chord $DE^2 = DG^2 + GE^2 = 416$,

and $DE^2 \div EG = \frac{416}{4} = 104$ the diameter of
the generating circle,

or the radius $OE = 52$,

hence $OI = 52 - 16 = 36$ the central distance,

and $HI^2 = OH^2 - OI^2 = 52^2 - 36^2 = 1408$,

$\frac{1}{3} DG^2 = \frac{1}{3}$ of 400 = $133\frac{1}{3}$

DG . . . $1274\frac{2}{3}$
20

$25493\frac{1}{3}$ 1st prod.
EG

$$GE \div 2 OE = \frac{4}{104} = \frac{1}{26} = .03846 \text{ a verfed line.}$$

Its tab. fegment	-	-	.00994
but 104^2 is	-	-	10816

43264
97344
97344

area of feg. DECGD	-	107.51104
mD \times mn = 12×40		480

gener. area mDECn	-	587.51104
OI	.	36

352506624
176253312

21150.39744	2d product
25493.33333	1st product

4342.93589	difference
2382.6	mult. inverted

260576

8686

3474

130

9

27287.5 answer

Ex. 2. What is the content of the middle frustum of a circular spindle, whose length is 20, greatest diameter 18, and least diameter 8?

Ans. 3657.1613.

PROBLEM XVIII.

To find the Superficies or Solidity of any Regular Body.

1. Multiply the proper tabular area (taken from the following table) by the square of the linear edge of the solid, for the superficies.

2. Multiply the tabular solidity by the cube of the linear edge, for the solid content.

Surfaces and Solidities of Regular Bodies.			
No. of faces	Names	Surfaces	Solidities
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icosaedron	8.66025	2.18169

EXAMPLES.

1. If the linear edge or side of a tetraedron be 3, required its surface and solidity?

The square of 3 is 9, and the cube 27. Then,

tab. surf. 1.73205 0.11785 tab. sol.

9

27

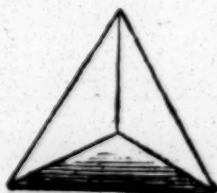
superf. 15.58845

82495

23570

solidity

3.18195



Ex.

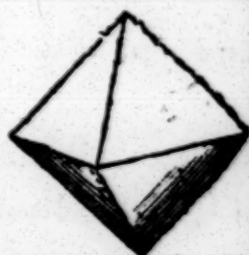
Ex. 2. What is the superficies and solidity of the hexaedron, whose linear side is 2?

$$\text{Ans. } \begin{cases} \text{superficies } 24 \\ \text{solidity } 8 \end{cases}$$



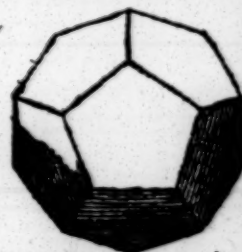
Ex. 3. Required the superficies and solidity of the octaedron, whose linear side is 2?

$$\text{Ans. } \begin{cases} \text{superficies } 13.85640 \\ \text{solidity } 3.77120 \end{cases}$$



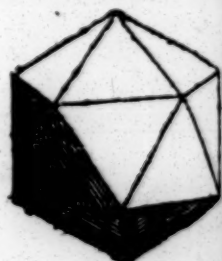
Ex. 4. What is the superficies and solidity of the dodecaedron, whose linear side is 2?

$$\text{An } \begin{cases} \text{superficies } 82.58292 \\ \text{solidity } 61.30496 \end{cases}$$



Ex. 5. Required the superficies and solidity of the icosaedron, whose linear side is 2?

$$\text{Ans. } \begin{cases} \text{superficies } 34.64100 \\ \text{solidity } 17.45352 \end{cases}$$



P R O.

PROBLEM XIX.

To find the Surface of a Cylindrical Ring.

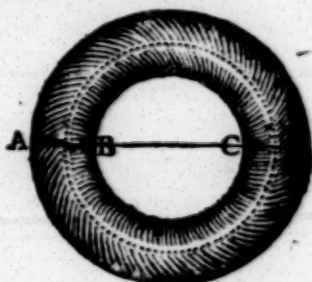
This figure being only a cylinder bent round into a ring, its surface and solidity may be found as in the cylinder, namely, by multiplying the axis, or length of the cylinder, by the circumference of the ring, or section, for the surface; and by the area of a section, for the solidity. Or use the following rules:

For the surface.—To the thickness of the ring add the inner diameter; multiply this sum by the thickness, and the product again by 9.8696, or the square of 3.1416.

EXAMPLES.

1. Required the superficies of a ring, whose thickness AB is 2 inches, and inner diameter BC is 12 inches?

12	9.8696
2	28
—	—
14	789568
2	197392
—	—
28	276.3488 ans.
—	—



- Ex. 2. What is the surface of the ring whose inner diameter is 16, and thickness 4? Ans. 789.568.

PROBLEM XX.

To find the Solidity of a Cylindrical Ring.

To the thickness of the ring add the inner diameter; then multiply that sum by the square of the thickness, and the product again by 2.4674, or $\frac{1}{4}$ of the square of 3.1416, for the solidity.

EX-

EXAMPLES.

1. Required the solidity of the ring, whose thickness is 2 inches, and its inner diameter 12?

$$\begin{array}{r}
 12 \\
 2 \\
 \hline
 14 \\
 4 \\
 \hline
 56
 \end{array}
 \qquad
 \begin{array}{r}
 2'4674 \\
 56 \\
 \hline
 148044 \\
 123370 \\
 \hline
 138'1744 \text{ answer}
 \end{array}$$

Ex. 2. What is the solidity of a cylindrical ring, whose thickness is 4, and inner diameter 16?

Ans. 789'568.

OF THE

CARPENTER'S RULE.

THIS instrument is otherwise called the sliding rule; and it is much used in measuring of timber and artificers' works, both for taking the dimensions, and computing the contents.

The instrument consists of two equal pieces, each a foot in length, which are connected together by a folding joint.

One side or face of the rule, is divided into inches, and half quarters or eighths. On the same face also are several plane scales, divided into 12th parts by diagonal lines; which are used in planning dimensions that are taken in feet and inches. The edge of the rule is commonly divided decimally, or into tenths; namely, each foot into 10 equal parts, and each of these into 10 parts again:

again: so that, by means of this last scale, dimensions are taken in feet and tenths and hundredths, and then multiplied as common decimal numbers, which is the best way:

On the one part of the other face are four lines, marked A, B, C, D; the two middle ones, B and C, being on a slider, which runs in a groove made in the stock. The same numbers serve for both these two middle lines, the one being above the numbers, and the other below.

These four lines are logarithmic ones, and the three A, B, C, which are all equal to one another, are double lines, as they proceed twice over from 1 to 10. The other or lowest line D, is a single one, proceeding from 4 to 40. It is also called the girt line, from its use in computing the contents of trees and timber. Upon it are also marked WG at 17.15, and AG at 18.75, the wine and ale gage points, to make this instrument serve the purpose of a gaging rule.

On the other part of this face there is a table of the value of a load, or 50 cubic feet, of timber, at all prices, from 6 pence to 2 shillings a-foot.

When 1 at the beginning of any line is accounted 1, or unit, then the 1 in the middle will be 10, and the 1 at the end 100; and when 1 at the beginning is accounted 10, then the 1 in the middle is 100, and the 1 at the end 1000; and so on. All the smaller divisions being altered proportionally.

PROBLEM I.

To multiply Numbers together.

Suppose the two numbers 13 and 24.—Set 1 on B to 13 on A; then against 24 on B stands 312 on A, which is the required product of the two given numbers 13 and 24.

Note. In any operations, when a number runs beyond the end of the line, seek it on the other radius, or other part of the line; that is, take the 10th part of it, or the

H

10th

100th part of it, &c. and increase the result proportionally 10 fold, or 100 fold, &c.

In like manner the product of 35 and 19 is 665.
and the product of 270 and 54 is 14580.

PROBLEM II.

To divide by the Sliding Rule.

As suppose to divide 312 by 24.—Set the divisor 24 on B to the dividend 312 on A; then against 1 on B stands 13, the quotient, on A.

Also 396 divided by 27 gives 14.6,
and 741 divided by 42 gives 17.6.

PROBLEM III.

To square any Number.

Suppose to square 23.—Set 1 on B to 23 on A; then against 23 on B, stands 529 on A, which is the square of 23.

Or, by the other two lines, set 1 or 100 on C to the 10 on D, then against every number on D, stands its square in the line C. So against 23 stands 529
against 20 stands 400
against 30 stands 900
and so on.

If the given number be hundreds, &c. reckon the 1 on D for 100, or 1000, &c. then the corresponding 1 on C is 10000, or 1000000, &c. So the square of 230 is found to be 52900.

PROBLEM IV.

To extract the Square Root.

Set 1 or 100, &c. on C to 1 or 10, &c. on D; then against every number found on C, stands its square root on D.

So,

So, against 529 stands its root 23
 against 400 stands its root 20
 against 900 stands its root 30
 against 300 stands its root 17.3
 and so on.

PROBLEM V.

To find a Mean Proportional between two Numbers.

As suppose between 29 and 430.—Set the one number 29 on C to the same on D; then against the other number 430 on C, stands their mean proportional 111 on D.

Also the mean between 29 and 320 is 96.3,
 and the mean between 71 and 274 is 139.

PROBLEM VI.

To find a Third Proportional to two Numbers.

Suppose to 21 and 32.—Set the first 21 on B to the second 32 on A; then against the second 32 on B, stands 48.8 on A; which is the third proportional sought.

Also the 3d. proportional to 17 and 29 is 49.4,
 and the 3d proportional to 73 and 14 is 2.5.

PROBLEM VII.

To find a Fourth Proportional to three Numbers.

Or, to perform the Rule-of-Three.

Suppose to find a fourth proportional to 12, 28, and 114.—Set the first term 12 on B to the 2d term 28 on A; then against the third term 114 on B, stands 266 on A, which is the fourth proportional sought.

Also the 4th proportional to 6, 14, 29, is 67.6,
 and the 4th proportional to 27, 20, 73, is 54.0.

TIMBER MEASURING.

PROBLEM L.

To find the Area, or Superficial Content, of a Board or Plank.

MULTIPLY the length by the mean breadth.

Note. When the board is tapering, add the breadths at the two ends together, and take half the sum for the mean breadth.

By the Sliding Rule.

Set 12 on B to the breadth in inches on A; then against the length in feet on B, is the content on A, in feet and fractional parts.

EXAMPLES.

1. What is the value of a plank, at $1\frac{1}{2}$ d. per foot, whose length is 12 feet 6 inches, and mean breadth 11 inches?

By Decimals.

$$\begin{array}{r} 12.5 \\ 11 \\ \hline 12 \mid 137.5 \\ 11.46 \\ 1\frac{1}{2} \text{d. is } \frac{1}{8} \mid 18 \text{ 5d ans.} \end{array}$$

By Duodecimals.

$$\begin{array}{r} 12 \quad 6 \\ 11 \\ \hline 1\frac{1}{2} \text{d. is } \frac{1}{8} \mid 11 \quad 5 \quad 6 \\ \phantom{1\frac{1}{2} \text{d. is } \frac{1}{8} \mid} \text{is } 4\frac{1}{2} \text{d} \\ 5 \text{ is } \mid 0 \quad \frac{1}{2} \\ \hline 18 \text{ 5d ans.} \end{array}$$

By the Sliding Rule.

As 12 B : 11 A :: $12\frac{1}{2}$ B : $11\frac{1}{2}$ A.

That is, as 12 on B is to 11 on A, so is $12\frac{1}{2}$ on B to $11\frac{1}{2}$ on A.

Ex.

Ex. 2. Required the content of a board, whose length is 11 feet 2 inches, and breadth 1 foot 10 inches.

Ans. 20^f 5ⁱ 8^u.

Ex. 3. What is the value of a plank, which is 12 feet 9 inches long, and 1 foot 3 inches broad, at 2 $\frac{1}{2}$ d. a foot?

Ans. 3s. 3 $\frac{1}{2}$ d.

Ex. 4. Required the value of 5 oaken planks at 3d per foot, each of them being 17 $\frac{1}{2}$ feet long: and their several breadths are as follow, namely, two of 13 $\frac{1}{2}$ inches in the middle, one of 14 $\frac{1}{4}$ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and 11 $\frac{1}{4}$ at the narrower.

Ans. £1 5 8 $\frac{1}{4}$.

PROBLEM II.

To find the Solid Content of Squared or Four-sided Timber.

Multiply the mean breadth by the mean thickness, and the product again by the length, and the last product will give the content.

By the Sliding Rule,

C D D C

As length : 12 or 10 :: quarter girt : solidity.

That is, as the length in feet on C, is to 12 on D when the quarter girt is in inches, or to 10 on D when it is in tenths of feet; so is the quarter girt on D, to the content on C.

Note 1. If the tree taper regularly from the one end to the other, either take the mean breadth and thickness in the middle, or take the dimensions at the two ends, and half their sum for the mean dimensions.

2. If the piece do not taper regularly, but is unequally thick in some parts and small in others; take several different dimensions, add them all together, and divide their sum by the number of them, for the mean dimensions.

3. The quarter girt is a geometrical mean proportional between the mean breadth and thickness, that is the square root of their product. Sometimes unskilful

measurers use the arithmetical mean instead of it, that is half their sum; but this is always attended with error, and the more so, as the breadth and depth differ the more from each other.

EXAMPLES.

1. The length of a piece of timber is 18 feet 6 inches, the breadths at the greater and less end 1 foot 6 inches and 1 foot 3 inches, and the thickness at the greater and less end 1 foot 3 inches and 1 foot: required the solid content.

Decimals.

$$\begin{array}{r}
 1.5 \\
 1.25 \\
 \hline
 2) 2.75 \\
 \underline{1.375} \\
 1.25 \\
 1.0 \\
 \hline
 2) 2.25 \\
 \underline{1.125} \\
 1.375 \\
 \hline
 5625 \\
 7875 \\
 3375 \\
 1125 \\
 \hline
 1546875 \\
 185 \\
 \hline
 7734375 \\
 12375000 \\
 1546875 \\
 \hline
 286171875
 \end{array}$$

mean breadth

mean depth
mean breadth

length

content

Duodecimals.

$$\begin{array}{r}
 1 \quad 6 \\
 1 \quad 3 \\
 \hline
 2) 2 \quad 9 \quad 6 \\
 \underline{1 \quad 4 \quad 6} \\
 1 \quad 3 \\
 1 \quad 0 \\
 \hline
 2) 2 \quad 3 \\
 \underline{1 \quad 1 \quad 6} \\
 1 \quad 1 \quad 6 \\
 4 \quad 6 \\
 6 \quad 9 \\
 \hline
 1 \quad 6 \quad 6 \quad 9 \\
 18 \quad 6 \\
 \hline
 27 \quad 10 \quad 1 \\
 9 \quad 3 \quad 4 \\
 \hline
 28 \quad 7 \quad 4 \quad 10 \\
 B_7
 \end{array}$$

By the Sliding Rule.

B A B A
 As 1 : $13\frac{1}{2}$:: $16\frac{1}{2}$: 223, the mean square.
 C D C D

As 1 : 1 :: 223 : 14.9, quarter girt,
 C D D C

As $18\frac{1}{2}$: 12 :: 14.9 : 28.6, the content.

Ex. 2. What is the content of the piece of timber, whose length is $24\frac{1}{2}$ feet, and the mean breadth and thickness each 1.04 feet?

Ans. $26\frac{1}{2}$ feet.

Ex. 3. Required the content of a piece of timber, whose length is 20.38 feet, and its ends unequal squares, the side of the greater being $19\frac{1}{8}$, and the side of the less $9\frac{7}{8}$ inches?

Ans. 29.756 feet.

Ex. 4. Required the content of the piece of timber, whose length is 27.36 feet; at the greater end the breadth is 1.78, and thickness 1.23; and at the less end the breadth is 1.04, and thickness 0.91?

Ans. 41.278 feet.

PROBLEM III.

To find the Solidity of Round or unsquared Timber.

Rule 1, or Common Rule.

Multiply the square of the quarter girt, or of $\frac{1}{4}$ of the mean circumference, by the length, for the content.

By the Sliding Rule.

As the length upon C : 12 or 10 upon D :: quarter girt, in 12ths or 10ths, on D : content on C.

Note, 1. When the tree is tapering, take the mean dimensions as in the former problems, either by girting it in the middle, for the mean girt, or at the two ends, and take half the sum of the two. But when the tree is very irregular, divide it into several lengths, and find the content of each part separately: or else, add all the girts together; and divide the sum by the number of them, for the mean girt.

2. This

2. This rule, which is commonly used, gives the answer about $\frac{1}{4}$ less than the true quantity in the tree, or nearly what the quantity would be after the tree is hewed square in the usual way: so that it seems intended to make an allowance for the squaring of the tree. When the true quantity is desired, use the 2d rule, given below.

EXAMPLES.

1. A piece of round timber being 9 feet 6 inches long, and its mean quarter girt 42 inches; what is the content?

<i>Decimals.</i>		<i>Duodecimals.</i>
3.5	quarter girt	3 6
3.5		3 6
<hr/>		<hr/>
175		10 6
105		1 9
<hr/>		<hr/>
12.25	length	12 3
9.5		9 6
<hr/>		<hr/>
6125		110 3
11025		6 1 6
<hr/>		<hr/>
116.375	content	116 4 6
<hr/>		<hr/>

By the Sliding Rule.

$$\begin{array}{cccc}
 C & D & D & C \\
 \text{As } 9.5 : 10 :: 35 : 116\frac{1}{3} \\
 \text{Or } 9.5 : 12 :: 42 : 116\frac{1}{3}
 \end{array}$$

Ex. 2. The length of a tree is 24 feet, its girt at the thicker end 14 feet, and at the smaller end 2 feet; required the content?

Ans. 96 feet.
Ex.

Ex. 3. What is the content of a tree, whose mean girt is 3'15 feet, and length 14 feet 6 inches?

Ans. 8'9922 feet.

Ex. 4. Required the content of a tree, whose length is 17 $\frac{1}{4}$ feet, which girts in five different places as follows, namely, in the first place 9'43 feet, in the second 7'92, in the third 6'15, in the fourth 4'74, and in the fifth 3'16?

Ans. 42'5195.

R U L E II.

Multiply the square of $\frac{1}{3}$ of the mean girt by double the length, and the product will be the content, very near the truth.

By the Sliding Rule.

As the double length on C : 12 or 10 on D :: $\frac{1}{3}$ of the girt, in 12ths or 10ths, on D : content on C.

E X A M P L E S.

1. What is the content of a tree, its length being 9 feet 6 inches, and its mean girt 14 feet?

<i>Decimals.</i>		<i>Duodecimals.</i>
2 ⁸	$\frac{1}{3}$ of girt	2 9 7
2 ⁸		2 9 7
<hr/>		<hr/>
224		5 7 2
56		2 1 3
<hr/>		<hr/>
7 ⁸⁴		1 8
19		<hr/>
<hr/>		7 10 1
7056		19
784		<hr/>
<hr/>		<hr/>
148 ⁹⁶	content	148 11 7

H 5

By

By the Sliding Rule.

C D D C
 As 19 : 10 :: 28 : 149.
 Or 19 : 12 :: 33 $\frac{6}{10}$: 149.

Ex. 2. Required the content of a tree, which is 24 feet long, and mean girt 8 feet? Ans. 122.88 feet.

Ex. 3. The length of a tree is 14 $\frac{1}{2}$ feet, and mean girt 3.15 feet; what is the content? Ans. 11.51 feet.

Ex. 4. The length of a tree is 17 $\frac{1}{4}$ feet, and its mean girt 6.28; what is the content? Ans. 54.4065 feet.

Other curious problems relating to the cutting of timber, so as to produce uncommon effects, may be found in my large Treatise on Mensuration.

ARTIFICERS' WORK.

ARTIFICERS compute the contents of their works by several different measures.

As glazing and masonry by the foot.

Painting, plastering, paving, &c. by the yard, of 9 square feet.

Flooring, partitioning, roofing, tiling &c. by the square, of 100 square feet.

And brickwork, either by the yard of 9 square feet, or by the perch, or square rod or pole, containing 272 $\frac{1}{4}$ square feet, or 30 $\frac{1}{4}$ square yards, being the square of the rod or pole, of 16 $\frac{1}{2}$ feet or 5 $\frac{1}{2}$ yards long.

As this number 272 $\frac{1}{4}$ is a troublesome number to divide by, the $\frac{1}{4}$ is often omitted in practice, and the content

tent in feet divided only by the 272. But when the exact divisor $272\frac{1}{4}$ is to be used, it will be easier to multiply the feet by 4, and then divide successively by 9, 11, and 11. Also to divide square yards by $30\frac{1}{4}$, first multiply them by 4, and then divide twice by 11.

All works, whether superficial or solid, are computed by the rules proper to the figure of them, whether it be a triangle, or rectangle, a parallelopiped, or any other figure.

BRICKLAYERS' WORK.

BRICKWORK is estimated at the rate of a brick and a half thick; so that if a wall be more or less than this standard thickness, it must be reduced to it, as follows:

Multiply the superficial content of the wall by the number of half bricks in the thickness, and divide the product by 3.

The dimensions of a building are usually taken by measuring half round on the outside and half round it on the inside; the sum of these two gives the compass of the wall, to be multiplied by the height for the content of the materials.

Chimneys are by some measured as if they were solid, deducting only the vacuity from the hearth to the mantle, on account of the trouble of them.

And by others they are girt or measured round for their breadth, and the height of the story is their height, taking the depth of the jambs for their thickness. And in this case no deduction is made for the vacuity from the floor to the mantle tree, because of the gathering of the breast and wings, to make room for the hearth in the next story.

To measure the chimney shafts, which appear above the building; girt them about with a line for the breadth, to multiply by their height. And account their thick-

ness half a brick more than it really is, in consideration of the plastering and scaffolding.

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed. But this deduction is made only with regard to materials; for the whole measure is taken for workmanship, and that all outside measure too, namely, measuring quite round the outside of the building, being in consideration of the trouble of the returns or angles. They have also some other allowances, such as double measure for feathered gable ends, &c.

EXAMPLES.

1. How many yards and rods of standard brickwork are in a wall whose length or compass is 57 feet 3 inches, and height 24 feet 6 inches; the walls being $2\frac{1}{2}$ bricks, or 5 half bricks thick?

<i>Decimals.</i>		<i>Duodecimals.</i>	
57 ²⁵		57	3
24 ⁵		24	6
<hr/>		<hr/>	
28625		234	0
22900		114	
11450		28	7 6
<hr/>		<hr/>	
1402 ⁶²⁵		1402	7 6
5 half bricks thick		<hr/>	
<hr/>		<hr/>	
3 7013 ¹²⁵		3 7013	1 6
9 2337 ⁷⁰⁸ $\frac{1}{2}$ sq. feet		9 2337	8 6
259 ⁷⁴⁵ $\frac{10}{27}$ yds.		2596	8 6
4		4	
<hr/>		<hr/>	
11 1038 ⁹⁸¹ $\frac{13}{27}$		11 1036	
11 94 ⁴⁵²⁸		11 94	2
rods 8 ⁵⁸⁶⁶ Anf.		8 r 17yds 6f 8 ⁶	
<hr/>		<hr/>	

By

By the Sliding Rule.

B A B A
As 1 : $24\frac{1}{2}$:: $57\frac{1}{4}$: 1403.

Ex. 2. Required the content of a wall 62 feet 6 inches long, and 14 feet 8 inches high, and $2\frac{1}{2}$ bricks thick?

Ans. 169.753 yards.

Ex. 3. A triangular gable is raised $17\frac{1}{2}$ feet high, on an end wall whose length is 24 feet 9 inches, the thickness being 2 bricks; required the reduced content?

Ans. $32.08\frac{1}{2}$ yards.

Ex. 4. The end wall of a house is 28 feet 10 inches long, and 55 feet 8 inches high to the eaves, 20 feet high is $2\frac{1}{2}$ bricks thick, other 20 feet high is 2 bricks thick, and the remaining 15 feet 8 inches is $1\frac{1}{2}$ brick thick, above which is a triangular gable of 1 brick thick, which rises 42 courses of bricks, of which every 4 courses make a foot. What is the whole content in standard measure?

Ans. 253.62 yards.

MASONS' WORK.

TO masonry belong all sorts of stone work; and the measure made use of is a foot, either superficial or solid.

Walls, columns, blocks of stone or marble, &c. are measured by the cubic foot; and pavements, slabs, chimney-pieces, &c. by the superficial or square foot.

Cubic or solid measure is used for the materials, and square measure for the workmanship.

In the solid measure, the true length, breadth, and thickness, are taken, and multiplied continually together. In the superficial, there must be taken the length and breadth of every part of the projection, which is seen without the general upright face of the building.

EX-

EXAMPLES.

1. Required the solid content of a wall, 53 feet 6 inches long, 12 feet 3 inches high, and 2 feet thick.

<i>Decimals.</i>		<i>Duodecimals.</i>
53'5		53 6
12 $\frac{3}{4}$		12 3
<hr/>		<hr/>
642'0		642 0
13'375		13 4 6
<hr/>		<hr/>
655'375		655 4 6
2		2
<hr/>		<hr/>
1310'750	Anf.	1310 9 0
<hr/>		<hr/>

By the Sliding Rule.

B	A	B	A
1	: 53 $\frac{1}{2}$: : 12 $\frac{3}{4}$: 655
1	: 655	: : 2	: 1310.

Ex. 2. What is the solid content of a wall, the length being 24 feet 3 inches, height 10 feet 9 inches, and 2 feet thick? Anf. 521'375 feet.

Ex. 3. Required the value of a marble slab, at 8s. per foot; the length being 5 feet 7 inches, and breadth 1 foot 10 inches. Anf. £4 1 10 $\frac{1}{2}$.

Ex. 4. In a chimney piece, suppose the length of the mantle and slab, each, 4f 6in

breadth of both together - 3 2

length of each jamb - - 4 4

breadth of both together - 1 9

Required the superficial content? Anf. 21f 10in.

C A R.

CARPENTERS

AND

JOINERS' WORK.

TO this branch belongs all the wood-work of a house, such as flooring, partitioning, roofing, &c.

Note. Large and plain articles are usually measured by the square foot or yard, &c. but enriched mouldings, and some other articles, are often estimated by running or lineal measure, and some things are rated by the piece.

In measuring of joists, it is to be observed, that only one of their dimensions is the same with that of the floor; and the other will exceed the length of the room by the thickness of the wall, and $\frac{1}{2}$ of the same, because each end is let into the wall about $\frac{1}{2}$ of its thickness.

No deductions are made for hearths, on account of the additional trouble and waste of materials.

Partitions are measured from wall to wall for one dimension, and from floor to floor, as far as they extend, for the other.

No deduction is made for door-ways, on account of the trouble of framing them.

In measuring of joiners' work, the string is made to ply close to every part of the work over which it passes.

The measure of centering for cellars is found by making a string pass over the surface of the arch for the one dimension, and taking the length of the cellar for the other; but in groin-centering, it is usual to allow double measure, on account of their extraordinary trouble.

In roofing, the length of the house in the inside, together with $\frac{1}{2}$ of the thickness of one gable, is to be considered as the length; and the breadth is equal to double the length of a string which is stretched from the ridge
down

down the rafter, and along the eaves-board, till it meets with the top of the wall.

For stair-cases, take the breadth of all the steps, by making a line ply close over them, from the top to the bottom, and multiply the length of this line by the length of a step for the whole area.—By the length of a step is meant the length of the front and the returns at the two ends; and by the breadth, is to be understood the girt of its two outer surfaces, or the tread and riser.

For the balustrade, take the whole length of the upper part of the hand-rail, and girt over its end till it meet the top of the newel post, for the length; and twice the length of the baluster upon the landing, with the girt of the hand-rail, for the breadth.

For wainscoting, take the compass of the room for the length; and the height from the floor to the cieling, making the string ply close into all the mouldings, for the breadth.—Out of this must be made deductions for windows, doors, and chimneys, &c. but workmanship is counted for the whole, on account of the extraordinary trouble.

For doors, it is usual to allow for their thickness, by adding it into both the dimensions of length and breadth, and then multiply them together for the area.—If the door be pannelled on both sides, take double its measure for the workmanship: but if one side only be pannelled, take the area and its half for the workmanship.—*For the surrounding architrave*, gird it about the outermost part for one dimension, and measure over it as far as it can be seen when the door is open, for the other.

Window-shutters, bases, &c. are measured in the same manner.

In the measuring of roofing for workmanship alone, the holes for chimney shafts and sky-lights are generally deducted.

But in measuring for work and materials, they commonly measure in all sky-lights, luthern-lights, and holes for the chimney shafts, on account of their trouble and waste of materials.

EX-

EXAMPLES.

1. Required the content of a floor 48 feet 6 inches long, and 24 feet 3 inches broad.

<i>Decimals.</i>		<i>Duodecimals.</i>	
48.5		48	6
24 $\frac{1}{4}$		24	3
<hr/>		<hr/>	
1940		204	0
970		96	
12.125		12	1 6
<hr/>		<hr/>	
1176.125	feet	1176	1 6
11.76125	Squares Ans.	11.76	1 6
<hr/>		<hr/>	

Ex. 2. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad, how many squares are in it?

Ans. 5 sq. 98 $\frac{1}{2}$ feet.

Ex. 3. How many squares are there in 173 feet 10 inches in length, and 10 feet 7 inches height, of partitioning?

Ans. 18.3972 squares.

Ex. 4. What cost the roofing of a house at 10s. 6d. a square; the length, within the walls, being 52 feet 8 inches, and the breadth 30 feet 6 inches; reckoning the roof $\frac{3}{4}$ of the flat?

Ans. £12 12 11 $\frac{1}{4}$.

Ex. 5. To how much, at 6s. per square yard, amounts the wainscoting of a room; the height, taking in the cornice and mouldings, being 12 feet 6 inches, and the whole compass 83 feet 8 inches; also the three window shutters are each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the door and shutters, being worked on both sides, are reckoned work and half work?

Ans. £36 12 2 $\frac{1}{2}$.

SLATERS

AND

TILERS' WORK.

IN these articles, the content of a roof is found by multiplying the length of the ridge by the girt over from eaves to eaves; making allowance in this girt for the double row of slates at the bottom, or for how much one row of slates or tiles is laid over another.

When the roof is of a true pitch, (that is, forming a right angle at top; then the breadth of the building with its half added, is the girt over both sides.

In angles formed in a roof, running from the ridge to the eaves, when the angle bends inwards, it is called a valley; but when outwards, it is called a hip.

Deductions are made for chimney shafts or window holes.

EXAMPLES.

1. Required the content of a slated roof, the length being 45 feet 9 inches, and the whole girt 34 feet 3 inches?

Decimals.

$$\begin{array}{r} 45.75 \\ 34.25 \\ \hline 18300 \\ 13725 \\ \hline 114375 \end{array}$$

$$\begin{array}{r} 9 \text{) } 1566.9375 \text{ feet} \\ \text{yds } 174.104 \end{array}$$

Answer

Duodecimals.

$$\begin{array}{r} 45 \quad 9 \\ 34 \quad 3 \\ \hline 205 \quad 6 \\ 135 \\ \hline 11 \quad 5 \quad 3 \\ \hline 9 \text{) } 1566 \quad 11 \quad 3 \\ 174^y \quad 11^i \quad 3'' \end{array}$$

Ex.

Ex. 2. To how much amounts the tiling of a house, at 25s 6d per square; the length being 43 feet 10 inches, and the breadth on the flat 27 feet 5 inches, also the eaves projecting 16 inches on each side, and the roof of a true pitch?

Ans. £24 9 5½.

PLASTERERS' WORK.

PLASTERERS' work is of two kinds, namely, **P**cieling, which is plastering upon laths; and rencering, which is plastering upon walls: which are measured separat ly.

The contents are estimated either by the foot or yard, or square of 100 feet. Inriched mouldings, &c. are rated by running or l neal measure.

Deductions are to be made for chimneys, doors, windows, &c. But the windows are seldom deducted, as the plastered returns at the top and sides are allowed to compensate for the window opening.

EXAMPLES.

1. How many yards contains the ceiling, which is 43 feet 3 inches long, and 25 feet 6 inches broad?

Decimals.		Duodecimals.	
43.25		43	3
25½		25	6
<hr/>		<hr/>	
21625		221	3
8650		86	
21625		21	7 6
<hr/>		<hr/>	
9) 1102.875		9) 1102	10 6
yds 122.541	Answer	122y	4 ^f 10 ⁱ 6 ^u
<hr/>		<hr/>	

1

Ex.

Ex. 2. To how much amounts the ceiling of a room, at 10d per yard; the length being 21 feet 8 inches, and the breadth 14 feet 10 inches? *Ans.* £1 9 8½.

Ex. 3. The length of a room is 18 feet 6 inches, the breadth 12 feet 3 inches, and height 10 feet 6 inches; to how much amounts the ceiling and rendering, the former at 8d and the latter at 3d per yard; allowing for the door of 7 feet by 3 feet 8, and a fire place of 5 feet square?

Ans. £1 13 3.

Ex. 4. Required the quantity of plastering in a room, the length being 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice, which girts 8½ inches, and projects 5 inches from the wall on the upper part next the ceiling: deducting only for a door 7 feet by 4.

Ans. 53⁷/₈ 5' 3" of rendering
 18 5 6 of ceiling
 39 0½ of cornice.

PAINTERS' WORK.

PAINTERS' work is computed in square yards. Every part is measured where the colour lies; and the measuring line is forced into all the mouldings, and corners.

Windows are done at so much a piece. And it is usual to allow double measure for carved mouldings, &c.

EXAMPLES.

1. How many yards of painting contains the room which is 65 feet 6 inches in compass, and 12 feet 4 inches high

Di-

Decimals.

$$\begin{array}{r} 65.5 \\ 12\frac{1}{2} \\ \hline \end{array}$$

786.0

21.83

$$\begin{array}{r} 9 \overline{) 807.83} \\ 89.7888 \end{array}$$
Duodecimals.

$$\begin{array}{r} 65 \quad 6 \\ 12 \quad 4 \\ \hline \end{array}$$

786 0

21 10

$$\begin{array}{r} 9 \overline{) 807 \quad 10} \\ \text{Answer} \quad 89 \quad 6 \quad 10 \end{array}$$

Ex. 2. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting are in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows each 6 feet by 3 feet 2 inches?

Ans. $73\frac{2}{7}$ yards.

Ex. 3. What cost the painting of a room, at 6d per yard; its length being 24 feet 6 inches, its breadth 16 feet 3 inches, and height 12 feet 9 inches; also the door is 7 feet by 3 feet 6, and the window shutters to two windows each 7 feet 9 by 3 feet 6, but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep: deducting the fire-place of 5 feet by 5 feet 6?

Ans. £3 3 10 $\frac{1}{2}$.

GLAZIERS' WORK.

GLAZIERS take their dimensions either in feet, inches and parts, or feet, tenths and hundredths. And they compute their work in square feet.

In taking the length and breadth of a window, the cross bars between the squares are included. Also windows of round or oval forms are measured as square, measuring them to their greatest length and breadth, on account of the waste in cutting the glass.

EX-

EXAMPLES.

1. How many square feet contains the window which is 4.25 feet long, and 2.75 feet broad?

Decimals.

$$\begin{array}{r} 2.75 \\ 4\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 11.00 \\ .6875 \\ \hline \end{array}$$

$$\begin{array}{r} 11.6875 \\ \hline \end{array}$$

Duodecimals.

$$\begin{array}{r} 2 \quad 9 \\ 4 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \quad 0 \\ \quad 8 \quad 3 \\ \hline \end{array}$$

$$\text{Answer } \begin{array}{r} 11 \quad 8 \quad 3 \\ \hline \end{array}$$

2. What will the glazing a triangular sky-light come to, at 10d per foot; the base being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches?

$$\text{Ans. } £1 \quad 15 \quad 1\frac{1}{2}$$

3. There is a house with three tier of windows, three windows in each tier, their common breadth 3 feet 11 inches;

now the height of the first tier is $7^f \quad 10^{in}$
of the second $6 \quad 8$
of the third $5 \quad 4$

Required the expence of glazing at 14d per foot?

$$\text{Ans. } £13 \quad 11 \quad 10\frac{1}{2}$$

4. Required the expence of glazing the windows of a house at 13d a foot; there being three stories, and three windows in each story:

the height of the lower tier is $7^f \quad 9^{in}$
of the middle - $6 \quad 6$
of the upper - $5 \quad 3\frac{1}{2}$

and of an oval window over the door $1 \quad 10\frac{1}{2}$

The common breadth of all the windows being 3 feet 9 inches.

$$\text{Ans. } £12 \quad 5 \quad 6$$

PAVERS' WORK.

PAVERS' work is done by the square yard. And the content is found by multiplying the length by the breadth.

EXAMPLES.

1. What cost the paving a foot-path at 3s 4d a-yard; the length being 35 feet 4 inches, and breadth 8 feet 3 inches?

<i>Decimals.</i>		<i>Duodecimals.</i>	
35 ⁴ / ₁₂		35	4
8 ³ / ₁₂		8	3
<hr/>		<hr/>	
282 ⁶⁶ / ₁₀₀		282	8
8 ⁸³ / ₁₀₀		8	10
<hr/>		<hr/>	
9) 291 ⁵ / ₁₀₀		9) 291	6
32 ³⁸ / ₁₀₀	Content	32	3 6
<hr/>		<hr/>	
2s is ¹ / ₁₀ 3 ²³ / ₁₀₀		£0	3 4
1s is ¹ / ₂₀ 1 ⁶¹ / ₁₀₀			4
4d is ¹ / ₃₀ 5 ³⁹ / ₁₀₀		<hr/>	
1 5 ³⁹ / ₁₀₀		0	13 4
20			8
<hr/>		<hr/>	
s 7 ⁹⁶ / ₁₀₀		5	6 8
12	3yd is ¹ / ₄	1	1 ¹ / ₄
<hr/>			2 ¹ / ₄
d 11 ⁵⁴ / ₁₀₀	Answer	£ 5	7 11 ¹ / ₂
<hr/>		<hr/>	

Ex. 2. What cost the paving a court, at 3s 2d per yard; the length being 27 feet 10 inches, and the breadth 14 feet 9 inches?

Ans. £ 7 4 5¹/₄.

EX-

Ex. 3. What will be the expence of paving a rectangular court yard, whose length is 63 feet, and breadth 45 feet; in which there is laid a foot-path of 5 feet 3 inches broad, running the whole length, with broad stones, at 3s a-yard; the rest being paved with pebbles at 2s 6d a yard?

Ans. £ 40 5 10½.

PLUMBERS' WORK.

P LUMBERS' work is rated at so much a-pound, or else by the hundred weight, of 112 pounds.

Sheet lead used in roofing, guttering, &c. is from 7 to 12lb to the square foot. And a pipe of an inch bore is commonly 13 or 14lb to the yard in length.

EXAMPLES.

1. How much weighs the lead which is 39 feet 6 inches long, and 3 feet 3 inches broad, at 8½lb to the square foot?

Decimals.

$$\begin{array}{r}
 39 \cdot 5 \\
 3 \frac{1}{4} \\
 \hline
 118 \cdot 5 \\
 9 \cdot 875 \\
 \hline
 128 \cdot 375 \\
 8 \frac{1}{2} \\
 \hline
 1027 \cdot 000 \\
 64 \cdot 1875 \\
 \hline
 1091 \cdot 1875
 \end{array}$$

Duodecimals.

$$\begin{array}{r}
 39 \quad 6 \\
 3 \quad 3 \\
 \hline
 118 \quad 6 \\
 9 \quad 10 \quad 6 \\
 \hline
 128 \quad 4 \quad 6 \\
 8 \frac{1}{2} \\
 \hline
 1024 \\
 64 \\
 2 \frac{3}{4} \\
 0 \frac{17}{8} \\
 \hline
 1091 \frac{9}{4} \text{ lb.}
 \end{array}$$

Answer

Ex 2. What cost the covering and guttering a roof with lead, at 18s the cwt.; the length of the roof being 43 feet, and breadth or girt over it 32 feet; the guttering 57 feet long, and 2 feet wide; the former 9·831lb, and the latter 7·373lb to the square foot?

Ans. £115 9 1½.

VAULTED

AND

ARCHED ROOFS.

ARCHED roofs are either vaults, domes, saloons, or groins.

Vaulted roofs are formed by arches springing from the opposite walls, and meeting in a line at the top.

Domes are made by arches springing from a circular or polygonal base, and meeting in a point at the top.

Saloons are formed by arches connecting the side walls to a flat roof, or ceiling, in the middle.

Groins are formed by the intersection of vaults with each other.

Vaulted roofs are commonly of the three following sorts:

1. *Circular roofs*, or those whose arch is some part of the circumference of a circle.

2. *Elliptical or oval roofs*, or those whose arch is an oval, or some part of the circumference of an ellipsis.

3. *Gothic roofs*, or those which are formed by two circular arcs, struck from different centres, and meeting in a point directly over the middle of the breadth, or span of the arch.

PROBLEM I.

To find the Surface of a Vaulted roof.

Multiply the length of the arch by the length of the vault, and the product will be the superficies.

1

Note.

Note. To find the length of the arch, make a line or string ply close to it, quite across from side to side.

EXAMPLES.

1. Required the surface of a vaulted roof, the length of the arch being 31·2 feet, and the length of the vault 120 feet?

$$\begin{array}{r} 31\cdot2 \\ 120 \\ \hline \end{array}$$

Ans. 3744·0 square feet.

Ex. 2. How many square yards are in the vaulted roof, whose arch is 42·4 feet, and the length of the vault 106 feet?

Ans. 499·37 yds.

PROBLEM II.

To find the Content of the Concavity of a Vaulted Roof.

Multiply the length of the vault by the area of one end, that is, by the area of a vertical transverse section, for the content.

Note. When the arch is an oval, multiply the span by the height, and the product by ·7854, for the area.

EXAMPLES.

1. Required the content of the concavity of a semi-circular vaulted roof, the span or diameter being 30 feet, and the length of the vault 150 feet?

$$\begin{array}{r} \cdot7854 \\ 900 \text{ the square of } 30 \\ \hline \end{array}$$

$$\begin{array}{r} 2) 706\cdot86 \\ 353\cdot43 \text{ area of the end} \\ 150 \text{ the length} \\ \hline \end{array}$$

$$\begin{array}{r} 1767150 \\ 35343 \\ \hline \end{array}$$

5301450 the content.

Ex.

Ex. 2. What is the content of the vacuity of an oval vault, whose span is 30 feet, and height 12 feet; the length of the vault being 60 feet? 6
 Anf. 1694'64.

Ex. 3. Required the content of the vacuity of a Gothic vault, whose span is 50 feet, the chord of each arch 50 feet, and the distance of each arch from the middle of these chords 10 feet; also the length of the vault 20.
 Anf. 35401'7.

PROBLEM III.

To find the Superficies of a Dome.

Find the area of the base, and double it; then say, as the radius of the base, is to the height of the dome, so is the double area of the base, to the superficies.

Note. For the superficies of a hemispherical dome, take the double area of the base only.

EXAMPLES.

1. To how much comes the painting of an octagonal spherical dome, at 8d per yard; each side of the base being 20 feet?

$$\begin{array}{r} 4'828427 \text{ tabular area} \\ 400 \text{ square of } 20 \\ \hline 1931'3708 \text{ area of the base} \\ 2 \end{array}$$

$$\begin{array}{r} 9) 3862'7416 \text{ superficies in feet} \\ 429'1934 \text{ yards} \\ 8 \end{array}$$

$$\begin{array}{r|l} 12 & 3433'5472 \\ 2,0 & 28'6 \\ \hline & 6'14 \end{array} \quad 1\frac{1}{2} \text{ answer.}$$

Ex. 2. Required the superficies of a hexagonal spherical dome, each side of the base being 10 feet.

$$\text{Anf. } 519'6152.$$

Ex. 3. What is the superficies of a dome with a circular base, whose circumference is 100 feet, and height 20 feet?

Ans. 2000 feet.

PROBLEM IV.

To find the Solid Content of a Dome.

Multiply the area of the base by $\frac{2}{3}$ of the height.

EXAMPLES.

i. Required the solid content of an octagonal dome, each side of the base being 20 feet, and the height 21 feet?

$$\begin{array}{r} 4828427 \\ 400 \end{array}$$

$$\begin{array}{r} 19313708 \text{ area of the base } \backslash \\ 14 \frac{2}{3} \text{ of height} \end{array}$$

$$\begin{array}{r} 77254832 \\ 19313708 \end{array}$$

270391912 answer.

Ex. 2. What is the solid content of a spherical dome, the diameter of whose circular base is 30 feet?

Ans. 7068.6 feet.

PROBLEM V.

To find the Superficies of a Saloon.

Find its breadth by applying a string close to it across the surface. Find also its length by measuring along the middle of it, quite round the room.

Then multiply these two together for the surface.

EXAMPLE.

The girt across the face of a saloon being 5 feet, and its mean compass about 100 feet, required the area or superficies?

$$\begin{array}{r} 100 \\ 5 \\ \hline 500 \text{ answer.} \end{array}$$

PROBLEM VI.

To find the Solid Content of a Saloon.

Multiply the area of a transverse section by the compass taken around the middle part. Subtract this product from the whole vacuity of the room, supposing the walls to go upright all the height to the flat ceiling. And the difference will be the answer.

EXAMPLE.

If the height AB of the saloon be 3.2 feet, the chord ADC of its front 1.5, and the distance DE of its middle part from the arch be 9 inches; required the solidity, supposing the mean compass to be 50 feet?

$$\begin{array}{r}
 2 \overline{) 4.5} \\
 \underline{2.25} \text{ AD} \\
 2.25 \\
 \underline{1125} \\
 450 \\
 \underline{450} \\
 5.0625 \text{ AD}^2 \\
 .5625 \text{ DE}^2 \\
 \hline
 5.6250 \quad (2.37 \text{ AE} \\
 4 \qquad \qquad 4 \\
 \hline
 43 \overline{) 162} \quad 3 \overline{) 9.48} \\
 \underline{3 \quad 129} \\
 33 \qquad \qquad 3.16 = \frac{4}{3} \text{ AE} \\
 \hline
 46 \overline{) 33} \qquad \qquad 4.50 \text{ AC} \\
 \hline
 7.66 \\
 \underline{.3} = \frac{4}{5} \text{ DE}
 \end{array}$$



2.298 area seg. ADCEA.

13

Again,

$$\begin{array}{r}
 \text{Again, } 3 \cdot 2 \quad 4 \cdot 5 \\
 \underline{3 \cdot 2} \quad \underline{4 \cdot 5} \\
 64 \quad 225 \\
 \underline{96} \quad \underline{180} \\
 10 \cdot 24 \quad 20 \cdot 25 \text{ AC}^2 \\
 \underline{\quad} \quad \underline{10 \cdot 24 \text{ AB}^2}
 \end{array}$$

$$\begin{array}{r}
 10 \cdot 01 \quad (3 \cdot 16 = \text{BC} \\
 \underline{9} \quad \underline{1 \cdot 6 = \frac{1}{2} \text{ AB}} \\
 61 \mid 1 \cdot 01 \quad 1896 \\
 1 \mid 61 \quad 316 \\
 \underline{\quad} \quad \underline{\quad} \\
 40 \quad 5 \cdot 056 \text{ area of triangle ABC} \\
 \underline{\quad} \quad \underline{2 \cdot 298 \text{ area seg.}} \\
 \quad \quad 2 \cdot 758 \text{ area of section AECBA} \\
 \quad \quad \underline{50 \text{ compass}} \\
 \underline{\quad} \quad 137 \cdot 900 \text{ content of the solid part.}
 \end{array}$$

Then this taken from the whole upright space, will leave the content of the vacuity contained within the room.

PROBLEM VII.

To find the Concave Superficies of a Groin.

To the area of the base add $\frac{1}{7}$ part of itself; for the superficial content.

EXAMPLES.

1. What is the superficial content of the groin arch, raised on a square base of 15 feet on each side?

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline \end{array}$$

$$\begin{array}{l} 7) \text{ 225 area of the base} \\ \quad 32\frac{1}{2} \text{ its 7th part} \\ \hline \quad 257\frac{1}{2} \text{ answer.} \\ \hline \end{array}$$

Ex. 2. Required the superficies of a groin arch, raised on a rectangular base, whose dimensions are 20 feet by 16.

Anf. $365\frac{1}{2}$.

PROBLEM VIII.

To find the Solid Content of a Groin Arch.

Multiply the area of the base by the height: from the product subtract $\frac{1}{8}$ of itself, and the remainder will be the content of the vacuity.

EXAMPLES.

1. Required the content of the vacuity within a groin arch, springing from the sides of a square base, each side of which is 16 feet.

$$\begin{array}{r} 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \text{ area of base} \\ \quad 8 \text{ height or radius} \\ \hline 2048 \\ 204\frac{4}{5} \text{ } \frac{1}{8} \text{ subtract} \\ \hline 1843\frac{1}{5} \text{ answer.} \end{array}$$

14

2. What

2. What is the content of a vacuity below an oval groin, the side of its square base being 24 feet, and its height 8 feet?

Ans. $4147\frac{1}{3}$.

NOTES.

1. To find the solid content of the brick or stone work, which forms any arch or vault: Multiply the area of the base by the height, including the work over the top of the arch; and from the product subtract the content of the vacuity, found by the foregoing problems; then the remainder will be the content of the solid materials.

2. In groin arches, however, it is usual to take the whole as solid, without deducting the vacuity, on account of the trouble and waste of materials, attending the cutting and fitting them to the arch.

LAND SURVEYING

CHAPTER I.

Description and use of the Instruments.

I. OF THE CHAIN.

LAND is measured with a chain, called Gunter's chain, of 4 poles or 22 yards in length, which consists of 100 equal links, the length of each link being $\frac{7\frac{1}{2}}{100}$ of a yard, or $\frac{66}{100}$ of a foot, or 7.92 inches, that is, nearly 8 inches or $\frac{2}{3}$ of a foot.

An acre of land is equal to 10 square chains, that is, 10 chains in length and 1 chain in breadth. Or it is 220×22 or 4840 square yards. Or it is 40×4 or 160 square poles. Or it is 1000×100 or 100000 square links. These being all the same quantity.

Also,

Also, an acre is divided into 4 parts called roods, and a rood into 40 parts called perches, which are square poles, or the square of a pole of $5\frac{1}{2}$ yards long, or the square of $\frac{1}{4}$ of a chain, or of 25 links, which is 625 square links. So that the divisions of land measure will be thus :

$$625 \text{ sq. links} = 1 \text{ pole or perch}$$

$$40 \text{ perches} = 1 \text{ rood}$$

$$4 \text{ roods} = 1 \text{ acre.}$$

The length of lines, measured with a chain, are best set down in links as integers, every chain in length being 100 links; and not in chains and decimals. Therefore after the content is found, it will be in square links; then cut off five of the figures on the right-hand for decima's, and the rest will be acres. Those decimals are then multiplied by 4 for roods, and the decimals of these again by 40 for perches.

EXAMPLE.

Suppose the length of a rectangular piece of ground be 792 links, and its breadth 385; to find the area in acres, roods, and perches.

$$\begin{array}{r}
 792 \\
 385 \\
 \hline
 3960 \\
 6336 \\
 2376 \\
 \hline
 304920
 \end{array}$$

ac ro p

Ans. 3 0 7

$$\begin{array}{r}
 4 \\
 \hline
 19680 \\
 40 \\
 \hline
 787200
 \end{array}$$

15

2. 07

2. OF THE PLAIN TABLE.

This instrument consists of a plane rectangular board of any convenient size, the centre of which, when used, is fixed by means of screws to a three-legged stand, having a ball and socket, or joint, at the top, by means of which, when the legs are fixed on the ground, the table is inclined in any direction.

To the table belong several parts : viz.

1. A frame of wood, made to fit round its edges, and to be taken off, for the convenience of putting a sheet of paper on the table. The one side of this frame is usually divided into equal parts, for drawing lines across the table, parallel or perpendicular to the sides ; and the other side of the frame is divided into 360 degrees, from a centre, which is in the middle of the table ; by means of which the table is to be used as a theodolite, &c.

2. A needle and compass screwed into the side of the table, or else in the middle of the support, to point out the directions ; and to be a check upon the sights.

3. An index, which is a brass two-foot scale, with either a small telescope, or open sights erected perpendicularly on the ends. These sights and one edge of the index are in the same plane, and that edge is called the fiducial edge of the index.

Before using this instrument, take a sheet of paper which will cover it, and wet it to make it expand ; then spread it flat on the table, pressing down the frame on the edges, to stretch it and keep it fixed there ; and when the paper is become dry, it will, by contracting again, stretch itself smooth and flat from any cramps and unevenness. On this paper is to be drawn the plan or form of the thing measured.

In using this instrument, begin at any part of the ground you think the most proper, and make a point on a convenient part of the paper or table, to represent that point of the ground ; then fix in that point one leg of the compasses, or a fine steel pin, and apply
to

to it the fiducial edge of the index, moving it round, till through the sights you perceive some remarkable object, as the corner of a field, &c. and from the station point draw a line with the point of the compasses along the fiducial edge of the index; then set another object or corner, and draw its line; do the same by another, and so on, till as many objects are set as may be thought necessary. Then measure from your station towards as many of the objects as may be necessary, and no more, taking the requisite offsets to corners or crooks in the hedges, &c. and lay the measures down on their respective lines on the table. Then, at any convenient place, measured to, fix the table in the same position, and set the objects which appear from thence, &c. as before; and thus continue till your work is finished, measuring such lines as are necessary, and determining as many as you can by intersecting lines of direction drawn from different stations.

And in these operations, observe the following particular cautions and directions:

1. Let the lines on which you make stations be directed towards objects as far distant as possible; and when you have set any such object, go round the table and look through the sights from the other end of the index, to see if any other remarkable object lie directly opposite; if there be not such an one, endeavour to find another forward object, such as shall have a remarkable backward opposite one, and make use of it rather than the other; because the back object will be of use in fixing the table in the original position, either when you have measured too near to the forward object, or when it may be hid from your sight at any necessary station by intervening hedges, &c.

2. Let the said lines, on which the stations are taken, be pursued as far as you conveniently can; for that will be the means of preserving more accuracy in the work.

3. At each station, it will be necessary to prove the truth of it; that is, whether the table be straight in

the line towards the object, and also whether the distance be rightly measured and laid down on the paper.—To know if the table be set down straight in the line; lay the index on the table in any manner, and move the table about, till through the sights you perceive either the fore or back object; then, without moving the table, go round it and look through the sights by the other end of the index, to see if the other object can be perceived; if it be, the table is in the line; if not, it must be shifted to one side, according to your judgment, till through the sights both objects can be seen.—The aforesaid operation only informs you if the station be straight in the line: but to know if it be in the right part of the line, that is, if the distance has been rightly laid down; fix the table in the original position, by laying the index along the station line, and turning the table about till the fore and back objects appear through the sights, and then also will the needle point at the same degree as at first; then lay the index over the station point and any other point on the paper representing an object which can be seen from the station; and if the said object appear straight through the sights, the station may be depended on as right; if not, the distance should be examined and corrected till the object can be so seen. And for this very useful purpose, it is advisable to have some high object or two, which can be seen from the greatest part of the ground, accurately laid down on the paper from the beginning of the survey, to serve continually as proof objects.

When, from any station, the fore and back objects cannot both be seen, the agreement of the needle with one of them may be depended on for placing the table straight on the line, and for fixing it in the original position.

Of shifting the Paper on the Plain Table.

When one paper is full, and you have occasion for more; draw a line in any manner through the farthest point of the last station line, to which the work can be
conve-

conveniently laid down; then take the sheet off the table, and fix another on, drawing a line on it, in a part the most convenient for the rest of the work; then fold or cut the old sheet by the line drawn on it, apply the edge to the line on the new sheet, and, as they lie in that position, continue the last station line on the new paper, placing on it the rest of the measure, beginning at where the old sheet left off. And so on from sheet to sheet.

When the work is done, and you would fasten all the sheets together into one piece, or rough plan, the afore-said lines are to be accurately joined together, as when the lines were transferred from the old sheets to the new ones.

But it is to be noted, that if the said joining lines, on the old and new sheet, have not the same inclination to the side of the table, the needle will not point to the original degree when the table is rectified; and if the needle be required to respect still the same degree of the compass, the easiest way of drawing the lines in the same position, is to draw them both parallel to the same sides of the table, by means of the equal divisions marked on the other two sides.

3. OF THE THEODOLITE.

The theodolite is a brazen circular ring, divided into 360 degrees, and having an index with sights, or a telescope, placed on the centre, about which the index is moveable; also a compass fixed to the centre, to point out courses and check the sights; the whole being fixed by the centre on a stand of a convenient height for use.

In using this instrument, an exact account, or field-book, of all measures and things necessary to be remarked in the plan, must be kept, from which to make out the plan on your return home from the ground.

Begin at such part of the ground, and measure in such directions, as you judge most convenient; taking angles or directions to objects, and measuring such distances as appear necessary, under the same restrictions as in
the

the use of the plain table. And it is safest to fix the theodolite in the original position at every station by means of fore and back objects, and the compass, exactly as in using the plain table; registering the number of degrees cut off by the index when directed to each object; and, at any station, placing the index at the same degree as when the direction towards that station was taken from the last preceding one, to fix the theodolite there in the original position, after the same manner as the plain table is fixed in the original position, by laying its index along the line of the last direction.

The best method of laying down the aforesaid lines of direction, is to describe a pretty large circle, quarter it, and lay on the circumference, the several numbers of degrees cut off by the index in each direction, marking the points they reach to; then, draw lines from the centre to all these points in the circumference; lastly, parallel to the said lines, draw other lines from station to station.

4. OF THE CROSS.

The cross consists of two pair of sights set at right angles to each other, on a staff having a sharp point at the bottom to stick in the ground.

The cross is very useful to measure small and crooked pieces of ground. The method is to measure a base or chief line, usually in the longest direction of the piece from corner to corner; and while measuring it, finding the places where perpendiculars would fall on this line, from the several corners and bends in the boundary of the piece, with the cross, by fixing it, by trials, on such parts of the line as that through one pair of the sights both ends of the line may appear, and through the other pair you can perceive the corresponding bends or corners; and then measuring the lengths of the said perpendiculars.

REMARKS.

Beside the fore-mentioned instruments, which are most commonly used, there are some others; as the circum-

ferentor, which resembles the theodolite in shape and use; and the semi-circle, for taking angles, &c. But of all the instruments for measuring, the plain table is certainly the best; not only because it may be used as a theodolite or semi-circle, by turning uppermost that side of the frame which has the 360 degrees on it; but because it is, in its own proper use, by much the easiest, safest, and most accurate for the purpose; for, by planning every part immediately on the spot, as soon as measured, there is not only saved a great deal of writing in the field-book, but every thing can also be planned more easily and accurately while it is in view, than it can afterwards from a field-book, in which many little things may be either neglected or mistaken; and besides, the opportunities which the plain table affords of correcting the work, or proving if it be right, at every station, are such advantages as can never be balanced by any other instrument. But although the plain table be the most generally useful instrument, it is not *always* so; there being many cases in which sometimes one instrument is the properest, and sometimes another; nor is that surveyor master of his business, who cannot in any case distinguish which is the fittest instrument or method, and use it accordingly: nay sometimes no instrument at all, but barely the chain itself is the best method, particularly in regular open fields lying together; and even when you are using the plain table, it is often of advantage to measure such large open parts with the chain only, and from those measures lay them down on the table.

The perambulator is used for measuring roads, and other great distances on level ground, and by the sides of rivers. It has a wheel of $8\frac{1}{4}$ feet, or half a pole in circumference, on which the machine turns; and the distance measured, is pointed out by an index, which is moved round by clock work.

Levels, with telescopic or other sights, are used to find the level between place and place, or how much one place is higher or lower than another. And in measuring any sloping or oblique line, either ascending or descending, a
small

small pocket level is useful for showing how many links for each chain are to be deducted, to reduce the line to the true horizontal length.

An offset staff is a very useful and necessary instrument, for measuring the offsets and other short distances. It is 10 links in length, being divided and marked at each of the 10 links.

Ten small arrows, or rods of iron or wood, are used to mark the end of every chain length, in measuring lines. And sometimes pickets, or staves with flags, are set up as marks or objects of direction.

Various scales are also used, in protracting and measuring on the plan or paper; such as plane scales, line of chords, protractor, compasses, reducing scale, parallel and perpendicular rules, &c. Of plane scales, there should be several sizes, as a chain in 1 inch, a chain in $\frac{3}{4}$ of an inch, a chain in $\frac{1}{2}$ an inch, &c. And of these, the best for use are those that are laid on the very edges of the ivory scale, to prick off distances by, without compasses.

THE FIELD-BOOK.

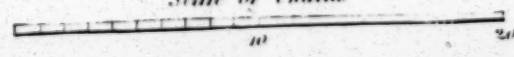
In surveying with the plain table, a field-book is not used, as every thing is drawn on the table immediately when it is measured. But in surveying with the theodolite, or any other instrument, some sort of a field-book must be used, to write down in it a register or account of all that is done and occurs relative to the survey in hand.

This book, every one contrives and rules as he thinks fittest for himself. The following is a specimen of a form very generally used. It is ruled into 3 columns: the middle, or principal column, is for the stations, angles, bearings, distances measured, &c.; and those on the right and left are for the offsets on the right and left, which are set against their corresponding distances in the middle column; as also for such remarks as may occur, and be proper to note in drawing the plan, &c.

Here \odot 1 is the first station, where the angle or bearing is $105^{\circ} 25'$. On the left, at 73 links in the distance

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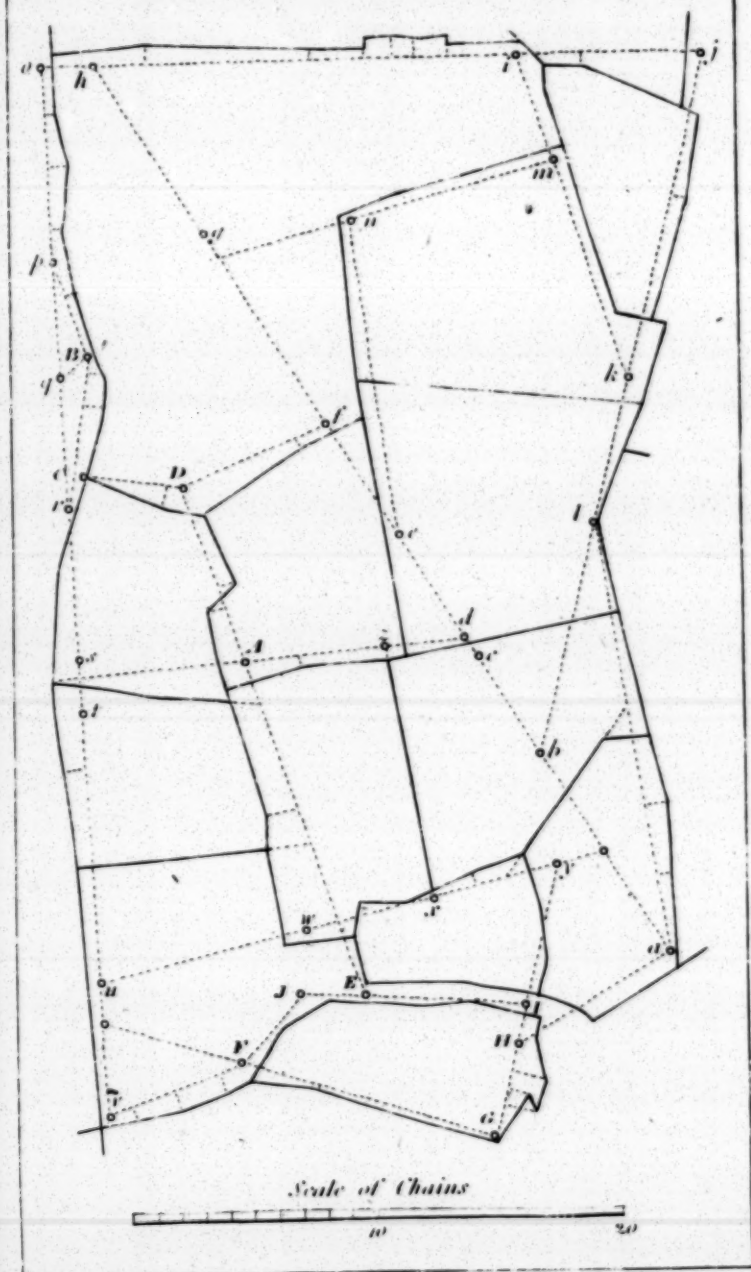
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Scale of Chains

Plan of the Field Book.

Plan from the foregoing Field Book.



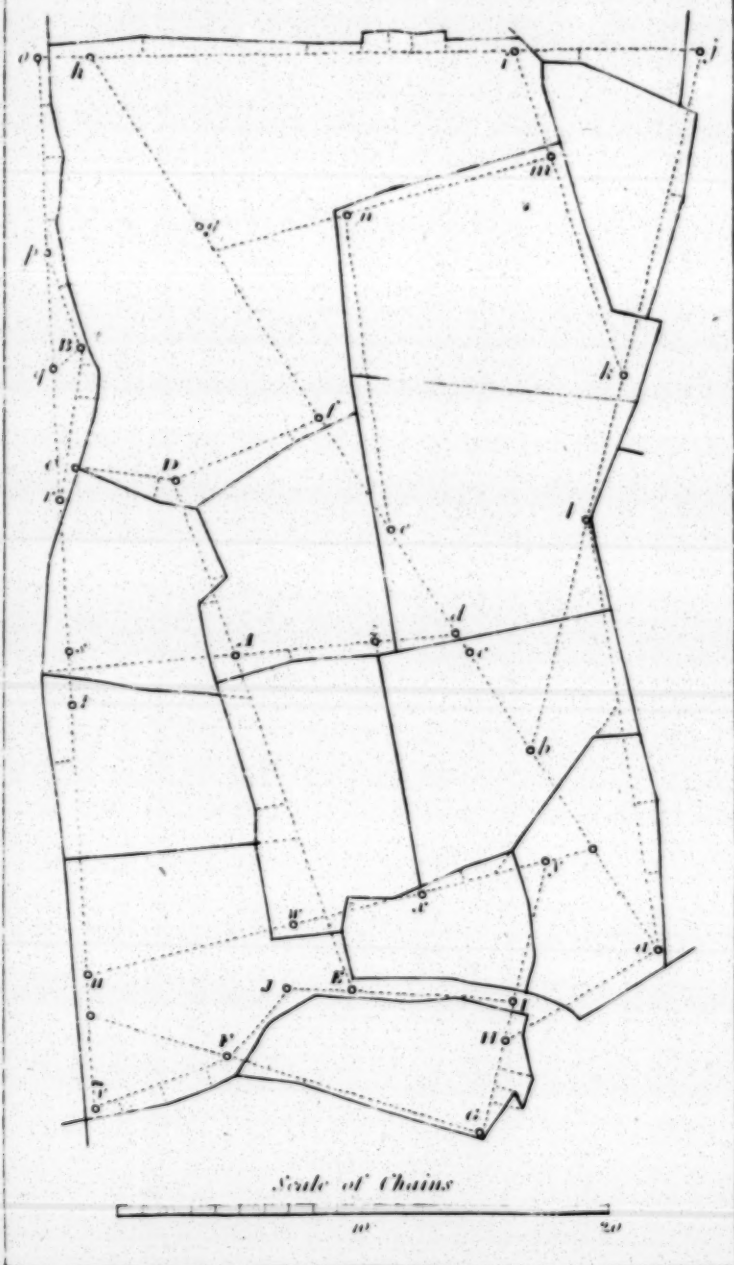
Field Book.

n		1370	1.56 to c
		836	56
		684	50
m		1480	90 to g
		960	21
		930	n
		700	48
		400	50
k		1430	to i
		1290	40
		1004	56
		980	m
		610	21
d		280	32
		1704	to l
		1464	22
	100	1050	32
		920	60
j		650	48
		350	14
		0	
		3074	to b
		2494	l
i		2100	
		2072	
		1730	
		1530	k
		1420	
h		1170	
		620	m
		32	
		280	j
		2574	
g		2494	
		2000	11
		1880	50
		1840	
		1794	i
f		1464	
		1328	
		1240	
		1130	
		800	
e		190	
		1450	h
		3570	g
		2020	f
		2590	
d		2240	
		2080	c
		1574	d
		1550	
		1510	c
a		990	b
		806	



Plan of the Field Book.

Plan from the foregoing Field Book.



Field Book.

n		1370	50 to e
		836	50
		684	50
m		1480	90 to g
		960	21
		930	n
		700	48
		400	50
k		1430	to i
		1290	40
		1004	50
		980	m
		610	21
		280	32
d		1794	to l
		1404	22
	100	1050	32
		920	60
		650	18
		350	41
		0	
j		3074	to b
		2494	l
		2100	
		2072	
		1750	
		1530	
		1120	k
		1170	
		620	to
		280	j
h		2574	
		2494	11
		2000	50
		1880	
		1840	
		1794	i
		1404	
		1328	
		1240	
		1130	
d		800	
		190	
		1150	h
		3570	g
		2020	f
		2590	
		2440	
d		2080	c
		1574	d
		1550	
		1540	c
		900	b

Field Book.

D		768	to 1
		326	70
		196	
		160	
		121	
		100	
C		435	D
		100	70
		18	to
B	56 44	600	to 1
		432	C
		160	
		36	
B		152	to q
p	24	480	B
		160	
d		1750	
		1600	11 to 5
		1028	
		930	A
		666	
		310	
		236	
		2418	180 to 6
		1950	
		1836	
u		1724	
		1600	
		1460	x
		1320	
		1110	
		1080	
		840	w
		750	50
		4440	36
		4120	
o		3884	u
		3380	60
		2992	90
		2692	t
		2624	
		2502	leave off
		2500	3
		2070	56
		1900	
		1840	r
		1770	
		1520	q
		808	p
		650	
		360	
		170	
h produce by i		220	o
		190	46

Field Book

<i>F</i>	40	560	to V
	70	500	
	70	500	
	70	100	
<i>J</i>	20	390	to F
		150	
<i>I</i>	15	954	<i>J</i>
		850	
	30	730	to E
		190	
	0	340	60
	20	280	50
<i>d</i>		170	
		744	to H
	70	672	0
	50	150	0
		15	
<i>G</i>		1160	to V
		1000	
		890	
		780	32
		590	10
		570	I
		530	10
		376	II
		256	150
		190	64
		144	50
			leave off
<i>do from u towards v</i>		1676	<i>G</i>
		1676	30
		896	24
		632	50
		620	50
		588	<i>F</i>
<i>D</i>		644	to F
		188	32
		2280	
		2270	<i>E</i>
	20	2230	
	56	2050	
		2030	
		1970	to W
		1552	130
		1380	96
		950	110
		860	54

Field Book.

		768	to J
		326	70
		196	
		160	
		124	
		100	
	70 40		
B		455	D
		100	76
		48	10
		600	to i
		432	C
	56	160	
	44	36	
B		152	to q
		480	B
		160	
	24		
		1150	
		1600	11 to s
		1028	
		930	A
		666	
		310	
		236	
	44 70 60		
d		2418	
		1950	160 ave off
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		1724	
		1600	
		1480	x
		1320	
		1110	
		1080	
		840	w
		750	50
		4440	36
		4120	
		3884	u
		3360	60
		2992	90
		2692	t
		2624	
		2592	
		2500	leave to W
		2070	36
		1900	
		1840	i
		1770	
		1520	q
		808	p
		650	
		360	
		170	
	leave off 40 80 20		
o			
	h produce by i	220	o
		190	46

or principal line, is an offset of 92; and at 610 an offset of 24 to a cross hedge. On the right, at 0, or the beginning, an offset 25 to the corner of the field; at 248 Brown's boundary hedge commences; at 610 an offset 35; and at 954, the end of the first line, the 0 denotes its terminating in the hedge. And so on for the other stations.

A line is drawn under the work, at the end of every station line, to prevent confusion.

Form of the Field-Book.

Offsets and Remarks on the left.	Stations, Bearings, and Distances.	Offsets and Remarks on the right.
	⊙ 1 105° 25'	
92	00	25 corner
cross a hedge 24	73	Brown's hedge
	248	
	610	35
	954	00
	⊙ 2 53° 10'	
house corner 51	00	00
	25	21
	120	29 a tree
34	734	40 a stile
	⊙ 3 67° 20'	
a brook 30	61	35
	248	
foot-path 16	639	16 a spring
cross hedge 18	810	
	973	20 a pound

But some skilful surveyors now make use of a different method for the field-book, namely, beginning at the bottom of the page, and writing upward; by which they sketch a neat boundary on either hand, as they pass along; an example of which will be given further on, in the method of surveying a large estate.

In smaller surveys and measurements, a good way of setting down the work, is, to draw, by the eye, on a piece of paper, a figure resembling that which is to be measured; and so writing the dimensions, as they are found, against the corresponding parts of the figure. And this method may be practised to a considerable extent, even in the larger surveys.

CHAPTER II.

THE PRACTICE OF SURVEYING.

THIS part contains the several works proper to be done in the field, or the ways of measuring by all the instruments, and in all situations.

PROBLEM I.

To measure a Line or Distance.

To measure a line on the ground with the chain: Having provided a chain, with 10 small arrows, or rods, to stick one into the ground, as a mark, at the end of every chain; two persons take hold of the chain, one at each end of it, and all the 10 arrows are taken by one of them, who is to go foremost, and is called the leader; the other being called the follower, for distinction's sake.

A picket, or station-staff, being set up in the direction of the line to be measured, if there do not appear some
marks

marks naturally in that direction; the follower stands at the beginning of the line, holding the ring at the end of the chain in his hand, while the leader drags forward the chain by the other end of it, till it is stretched straight, and laid or held level, and the leader directed, by the follower waving his hand, to the right or left, till the follower see him exactly in a line with the mark or direction to be measured to; there both of them stretching the chain straight, and stooping and holding it level, the leader having the head of one of his arrows in the same hand by which he holds the end of the chain, he there sticks one of them down with it while he holds the chain stretched. This done, he leaves the arrow in the ground, as a mark for the follower to come to, and advances another chain forward, being directed in his position by the follower, standing at the arrow, as before; as also by himself now, and at every succeeding chain's length, by moving himself from side to side, till he brings the follower and the back mark into a line. Having then stretched the chain, and stuck down an arrow, as before, the follower takes up his arrow, and they advance again in the same manner another chain-length. And thus they proceed, till all the 10 arrows are employed, and are in the hands of the follower; and the leader, without an arrow, is arrived at the end of the 11th chain-length. The follower then sends or brings the 10 arrows to the leader, who puts one of them down at the end of his chain, and advances with the chain as before. And thus the arrows are changed from the one to the other at every 10 chains' length, till the whole line is finished; then the number of changes of the arrows shews the number of tens, to which the follower adds the arrows he holds in his hand, and the number of links of another chain over to the mark or end of the line. So if there have been 3 changes of the arrows, and the follower hold 6 arrows, and the end of the line cut off 45 links more, the whole length of the line is set down in links thus, 3645.

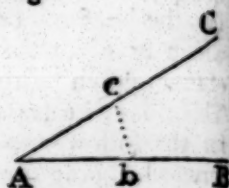
When the ground is on a declivity, ascending or descending;

scending; at every chain length, lay the offset-staff, or link-staff down in the slope of the chain, on which lay the small pocket level, to shew how many links or parts the slope line is longer than the true level one; then draw the chain forward so many links or parts, which reduces the line horizontal.

PROBLEM II.

To take Angles and Bearings.

Let B and C be two objects, or two pickets set up perpendicular, and let it be required to take their bearings, or the angle formed between them at any station A.



1. *With the Plain Table,*

The table being covered with a paper, and fixed on its stand; plant it at the station A, and fix a fine pin, or a point of the compasses in a proper point of the paper, to represent the point A: Close by the side of this pin lay the fiducial edge of the index, and turn it about, still touching the pin till one object B can be seen through the sights: then by the fiducial edge of the index draw a line. In the very same manner draw another line in the direction of the other object C. And it is done.

2. *With the Theodolite, &c.*

Direct the fixed sights along one of the lines, as AB, by turning the instrument about till you see the mark B through these sights; and there screw the instrument fast. Then turn the moveable index about till, through its sights, you see the other mark C. Then the degrees cut by the index, on the graduated limb or ring of the instrument, shew the quantity of the angle.

3. *With the Magnetic Needle and Compass.*

Turn the instrument, or compass, so, that the north end of the needle point to the flower-de-luce. Then direct

direct the sights to one mark as B, and note the degrees cut by the needle. Next direct the sights to the other mark C, and note again the degrees cut by the needle. Then their sum or difference, as the case is, will give the quantity of the angle BAC.

4. *By Measurement with the Chain, &c.*

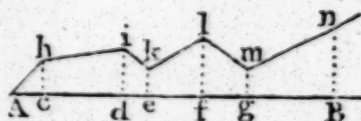
Measure one chain length, or any other length, along both directions, as to b and c. Then measure the distance b c, and it is done.—This is easily transferred to paper, by making a triangle A b c with these three lengths, and then measuring the angle A as in Practical Geometry, prob. xi.

PROBLEM III.

To measure the Offsets.

A h i k l m n being a crooked hedge, or river, &c. From A measure in a straight direction along the side of it to B. And in measuring along this line A B, observe when you are directly opposite any bends or corners of the hedge, as at c, d, e, &c. and from thence measure the perpendicular offsets c h, d i, &c. with the offset-staff, if they are not very large, otherwise with the chain itself. And the work is done. The register, or field-book of which, may be as follows:

Offs. left.	Base line A B.	
	o	⊙ A
ch	62	45 A c
di	84	220 A d
ek	70	340 A e
fl	88	510 A f
gm	57	634 A g
B n	91	785 A B



Note. When the offsets are not very large, their places c, d, e, &c. on the base line, can be very well determined

determined by the eye, especially when assisted by laying down the offset-staff in a cross or perpendicular direction. But when these perpendiculars are very large, find their positions by the cross, or by the instrument which you happen to be using, in this manner: As you measure along A B, when you come nearly opposite C, where you judge a perpendicular will stand, plant your instrument in the line, and turn the index till the marks A and B can be seen through both the sights, looking both backward and forward; then look along the cross sights, or the cross line on the index; and if it point directly to the corner or bend h, the place of c is right. Otherwise, move the instrument backward or forward on the line A B, till the cross line points straight to h. This being found, set down the distance measured from A to c: then measure the offset c h, and set it down opposite the former, and on the left hand side.

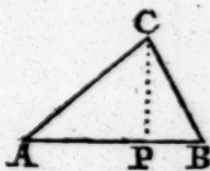
Then proceed forward in the line A B, till you arrive opposite another corner, and determine the place of the perpendicular as before. And so on throughout the whole length.

PROBLEM IV.

To survey a Triangular Field ABC.

1. *By the Chain.*

AP 794
AB 1321
PC 826



Having set up marks at the corners, which is to be done in all cases where there are not marks naturally; measure with the chain from A to P, where a perpendicular would fall from the angle C, and set up a mark at P, noting down the distance AP. Then complete the distance

distance AB by measuring from P to B. Having set down this measure, return to P, and measure the perpendicular PC. And thus, having the base and perpendicular, the area from them is easily found. Or having the place P of the perpendicular, the triangle is easily constructed.

Or, measure all the three sides with the chain, and note them down. From which the content is easily found, or the figure constructed.

2. *By taking one or more of the Angles.*

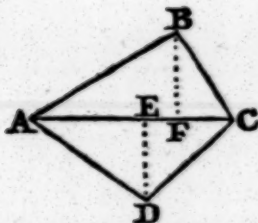
Measure two sides AB, AC, and the angle A between them. Or measure one side AB, and the two adjacent angles A and B. From either of these ways the figure is easily planned; then by measuring the perpendicular CP on the plan, and multiplying it by half AB, you have the content.

PROBLEM V.

To measure a Four-sided Field.

1. *By the Chain.*

AE	214	210	DE
AF	362	360	BF
AC	592		

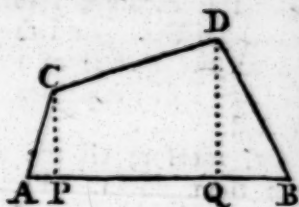


Measure along either of the diagonals, as AC; and either the two perpendiculars DE, BF, as in the last problem; or else the sides AB, BC, CD, DA. From either of these ways the figure may be planned and computed, as before directed

Other.

Otherwise by the Chain.

AP	110	352	PC
AQ	745	595	QD
AB	1110		



Measure on the longest side, the distances AP, AQ, AB; and the perpendiculars PC, QD.

2. *By taking one or more of the Angles.*

Measure the diagonal AC (see the last fig. but one,) and the angles CAB, CAD, ACB, ACD.—Or measure the four sides, and any one of the angles as BAD.

Thus		Or thus	
AC	591	AB	486
CAB	37°20'	BC	394
CAD	41 15	CD	410
ACB	72 25	DA	462
ACD	54 40	BAD	78°35'

PROBLEM VI.

To survey any Field by the Chain only.

Having set up marks at the corners, where necessary, of the proposed field ABCDEFG. Walk over the ground, and consider how it can best be divided in triangles and trapeziums; and measure them separately as in the last two problems. And in this way it will be proper to divide it into triangles and trapeziums, by drawing diagonals from corner to corner; and so as that all the perpendiculars may fall within the figures. Thus, the following figure is divided into the two trapeziums ABCG, GDEF, and the triangle GCD. Then, in the first trapezium, beginning at A, measure the diagonal AC,

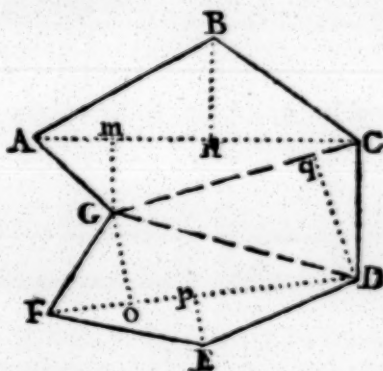
and the two perpendiculars Gm , Bn . Then the base GC , and the perpendicular Dq . Lastly, the diagonal DF , and the two perpendiculars pE , oG . All which measures write against the corresponding parts of a rough figure, drawn to resemble the figure to be surveyed, or set them down in any other form you choose.

Thus

Am 135	$130\ m\ G$
An 410	$180\ n\ B$
Ac 550	

Cq 152	$20q\ D$
CG 440	

Fo 206	$120\ o\ G$
Fp 288	$80\ p\ E$
FD 520	



Or Thus.

Measure all the sides AB , BC , CD , DE , EE , FG , and GA ; and the diagonals AC , CG , GD , DF .

Otherwise.

Many pieces of land may be very well surveyed, by measuring any base line, either within or without them, together with the perpendiculars let fall on it from every corner of them. For they are by those means divided into several triangles, and trapezoids, all whose parallel sides are perpendicular to the base line; and the sum of these triangles and trapeziums will be equal to the figure proposed if the base line fall within it; if not, the sum of the parts which are without being taken from the sum of the whole which are both within and without, will leave the area of the figure proposed.

In pieces that are not very large, it will be sufficiently exact to find the points, in the base line, where the several perpendiculars will fall, by means of the *Cross*, and

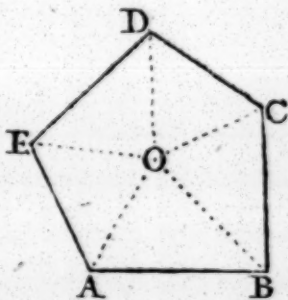
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measure the distance to E, laying it on the line from C to E. In like manner determine the positions of CA and CB, by turning the sights successively to A and B; and lay the lengths of those lines down. Then connect the points with the boundaries of the field, by drawing the black lines CD, DE, EA, AB, BC.

2. *From a Station Within the Field.*

When all the other parts cannot be seen from one angle, choose some place O within; or even without, if more convenient; from whence the other parts can be seen. Plant the table at O, then fix it with the needle north, and mark the point O on it. Apply the index successively to O, turning it round with the sights to each angle A, B, C, D, E, drawing dry lines to them along the edge of the index; then measuring the distances OA, OB, &c. and laying them down on those lines. Lastly, draw the boundaries AB, BC, CD, DE, EA.



3. *By going Round the Figure.*

When the figure is a wood, or water, or when from some other obstruction you cannot measure lines across it; begin at any point A, and measure round it, either within or without the figure, and draw the directions of all the sides, thus: Plant the table at A, turn it with the needle to the north or flower-de-luce, fix it, and mark the point A. Apply the index to A, turning it till you can see the point E, there draw a line; and then the point B, and there draw a line: then measure these lines, and lay them down from A to E and B. Next move the table to B, lay the index along the line AB, and turn the table about

till you can see the mark A, and screw fast the table; in which position also the needle will again point to the flower-de-luce, as it will do indeed at every station when the table is in the right position. Here turn the index about B till through the sights you see the mark C; there draw a line, measure BC, and lay the distance on that line after you have set down the table at C. Turn it then again into its proper position, and in like manner find the next line CD. And so on, quite round by E, to A again. Then the proof of the work will be the joining at A: for if the work be all right, the last direction EA on the ground, will pass exactly through the point A on the paper; and the measured distance will also reach exactly to A. If these do not coincide, or nearly so, some error has been committed, and the work must be examined over again.

PROBLEM VIII.

To survey a Field with the Theodolite, &c.

1. *From One Point or Station.*

When all the angles can be seen from one point, as the angle C, (first fig. to last prob.) ; place the instrument at C, and turn it about till, through the fixed sights, you see the mark B, and there fix it. Then turn the moveable index about, till the mark A is seen through the sights, and note the degrees cut on the instrument. Next turn the index successively to E and D, noting the degrees cut off at each; which gives all the angles BCA, BCE, BCD. Lastly, measure the lines CB, CA, CE, CD; and enter the measures in a field-book, or rather against the corresponding parts of a rough figure, drawn by guess, to resemble the field.

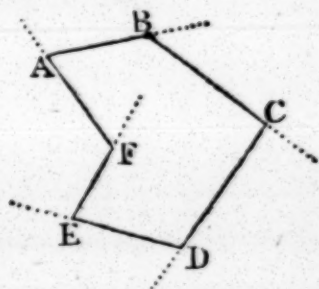
2. *From a Point Within or Without.*

Plant the instrument at O, (last fig.) and turn it about till the fixed sights point to any object as A; and there screw it fast. Then turn the moveable index round, till the

the sights point successively to the other points E, D, C, B, noting the degrees cut off at each of them; which gives all the angles round the point O. Lastly, measure the distances OA, OB, OC, OD, OE, noting them down as before, and the work is done.

3. *By going Round the Field.*

By measuring round, either within or without the field, proceed thus. Having set up marks at B, C, &c. near the corners as usual, plant the instrument at any point A, and turn it till the fixed index be in the direction AB, and there screw it fast:



then turn the moveable index to the direction AF; and the degrees cut off will be the angle A. Measure the line AB, and plant the instrument at B, and there in the same manner observe the angle B. Then measure BC, and observe the angle C. Then measure the distance CD, and take the angle D. Then measure DE, and take the angle E. Then measure EF, and take the angle F. And lastly measure the distance FA.

To prove the work; add all the inward angles A, B, C, &c. together, and when the work is right, their sum will be equal to twice as many right angles, as the figure has sides, wanting four right angles. And when there is an angle, as F, that bends inwards, and you measure the external angle, which is less than two right angles, subtract it from four right angles, or 360 degrees, to give the internal angle greater than a semicircle or 180 degrees.

Otherwise.

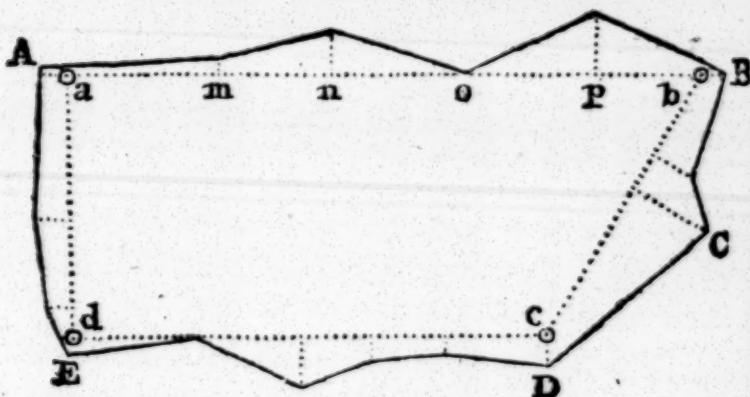
Instead of observing the internal angles, you may take the external angles, formed without the figure by producing the sides further out. And in this case, when the

work is right, their sum altogether will be equal to 360 degrees. But when one of them, as F, runs inwards, subtract it from the sum of the rest, to leave 360 degrees.

PROBLEM IX.

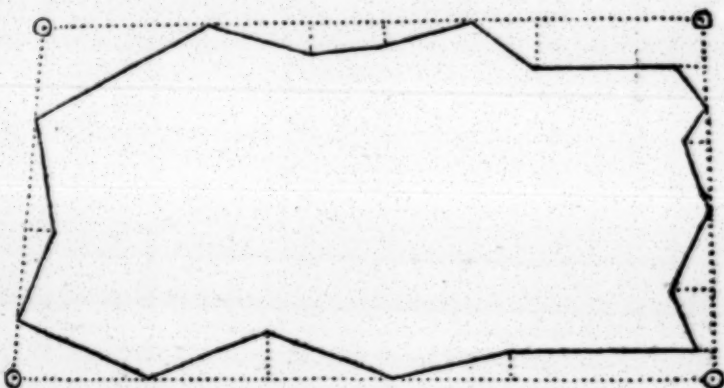
To survey a Field with Crooked Hedges, &c.

With any of the instruments measure the lengths and positions of imaginary lines running as near the sides of the field as you can; and, in going along them, measure the offsets in the manner before taught; and you will have the plan on the paper in using the plain table, drawing the crooked hedges through the ends of the offsets; but in surveying with the theodolite, or other instrument, set down the measures properly in a field-book, or memorandum book, and plan them after returning from the field, by laying down all the lines and angles.



So, in surveying the piece ABCDE, set up marks a, b, c, d, dividing it into as few sides as may be, commonly four. Then begin at any station a, and measure the lines ab, bc, cd, da, and take their positions, or the angles a, b, c, d; and, in going along the lines, measure all the offsets, as at m, n, o, p, &c. along every station line.
And

And this is done either within the field, or without, as may be most convenient. When there are obstructions within, as wood, water, hills, &c. then measure without, as in the figure here below.



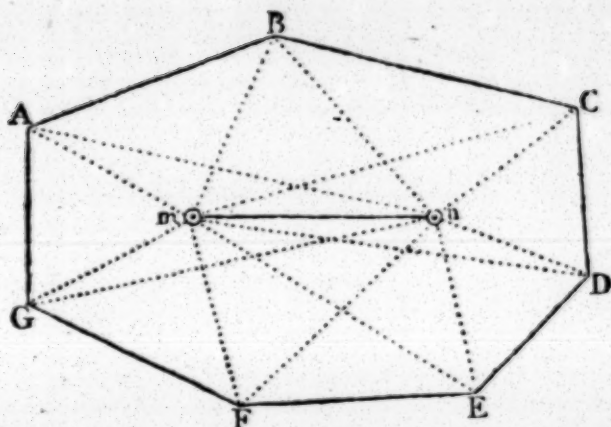
PROBLEM X.

To survey a Field, or any other Thing, by Two Stations.

This is performed by choosing two stations, from whence all the marks and objects can be seen, then measuring the distance between the stations, and at each station taking the angles formed by every object, from the station line or distance.

The two stations may be taken either within the bounds, or in one of the sides, or in the direction of two of the objects, or quite at a distance and without the bounds of the objects, or part to be surveyed.

In this manner, not only grounds may be surveyed, without even entering them, but a map may be taken of the principal parts of a county, or the chief places of a town, or any part of a river or coast surveyed, or any other inaccessible objects; by taking two stations, on two towers, or two hills, or such like.



When the plain table is used; plant it at one station *m*, draw a line *m n* on it, along which lay the edge of the index, and turn the table about till the sights point directly to the other station; and there screw it fast. Then turn the sights round *m* successively to all the objects *A, B, C, &c.* drawing a dry line by the edge of the index at each, as *m A, m B, m C, &c.* Then measure the distance to the other station, there plant the table, and lay that distance down on the station line from *m* to *n*. Next lay the index by the line *n m*, and turn the table about till the sights point to the other station *m*, and there screw it fast. Then direct the sights successively to all the objects *A, B, C, &c.* as before, drawing lines each time, as *n A, n B, n C, &c.* and their intersection with the former lines, will give the places of all the objects, or corners, *A, B, C, &c.*

When the theodolite, or any other instrument for taking angles, is used; proceed in the same way, measuring the station distance *m n*, planting the instrument first at one station and then at another; then placing the fixed sights in the direction *m n*, and directing the moveable sights to every object, noting the degrees cut off at each time. Then, these observations being planned, the intersections of the lines will give the objects as before.

When

When all the objects to be surveyed cannot be seen from two stations ; then three stations may be used, or four, or as many as necessary ; measuring always the distance from one station to another ; placing the instrument in the same position at every station, by means described before ; and from each station observing or setting every object that can be seen from it, by taking its direction or angular position, till every object be determined by the intersection of two or more lines of direction, the more the better. And thus may very extensive surveys be taken, as of large commons, rivers, coasts, countries, hilly grounds, and such like.

PROBLEM XI.

To survey a Large Estate.

If the estate be very large, and contain a great number of fields, it cannot well be done by surveying all the fields singly, and then putting them together ; nor can it be done by taking all the angles and boundaries that inclose it. For in these cases, any small errors will be so multiplied, as to render it very much distorted.

1. Walk over the estate two or three times, in order to get a perfect idea of it, and till you can carry the map of it tolerably in your head. And to help your memory, draw an eye draught of it on paper, or at least of the principal parts of it, to guide you ; setting the names within the fields in that draught.

2. Choose two or more eminent places in the estate, for your stations, from whence you can see all the principal parts of it : and let these stations be as far distant from one another as possible ; as the fewer stations you have to command the whole, the more exact your work will be ; and they will be fitter for your purpose, if these station lines be in or near the boundaries of the ground, especially if two lines or more proceed from one station.

3. Take what angles, between the stations, you think necessary, and measure the distances from station to station,

always in a right line: these things must be done, till you get as many angles and lines as are sufficient for determining all your points of station. And in measuring any of these station distances, mark accurately where these lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by measuring its distance from the station line; and where a perpendicular from it cuts that line; and always mind, in any of these observations, that you be in a right-line, which you will know by taking a backsight and foresight, along your station line. And thus as you go along any main station line, take offsets to the ends of all hedges, and to any pond, house, mill, bridge, &c. omitting nothing that is remarkable. And all these things must be noted down: for these are your data, by which the places of such objects are to be determined on the plan. And be sure to set marks up at the intersections of all hedges with the station line, that you may know where to measure from; when you come to survey these particular fields, which must immediately be done, as soon as you have measured that station line, while they are fresh in memory. In this way all your station lines are to be measured, and the situation of all places adjoining to them determined, which is the first grand point to be obtained. It will be proper for you to lay down the work on paper every night, when you go home, that you may see how you go on.

4. As to the inner parts of the estate, they must be determined in like manner, by new station lines: for, after the main stations are determined, and every thing adjoining to them, then the estate must be subdivided into two or three parts by new station lines; taking inner stations at proper places, where you can have the best view. Measure these station lines as you did the first, and all their intersections with hedges, and all offsets to such objects as appear. Then you may proceed to survey the adjoining fields, by taking the angles that the sides make with the station line, at the intersections, and measuring the distances to each corner, from the intersections. For the

station lines will be the bases to all the future operations; the situation of all parts being entirely dependent on them: and therefore they should be taken of as great length as possible; and it is best for them to run along some of the hedges or boundaries of one or more fields, or to pass through some of their angles. All things being determined for these stations, you must take more inner stations, and continue to divide and subdivide till at last you come to single fields; repeating the same work for the inner stations, as for the outer ones, till all be done; and close the work as often as you can, and in as few lines as possible. And that you may choose stations the most conveniently, so as to cause the least labour, let the station lines run as far as you can along some hedges, and through as many corners of the fields, and other remarkable points, as you can. And take notice how one field lies by another; that you may not misplace them in the draught.

5. An estate may be so situated, that the whole cannot be surveyed together; because one part of the estate cannot be seen from another. In this case you may divide it into three or four parts, and survey the parts separately, as if they were lands belonging to different persons; and at last join them together.

6. As it is necessary to protract or lay down the work as you proceed in it, you must have a scale of a due length to do it by. To get such a scale, measure the whole length of the estate in chains; then consider how many inches long the map is to be; and from these you will know how many chains you must have in an inch; then make your scale accordingly, or choose one already made.

7. The trees in every hedge row may be placed in their proper situation, which is soon done by the plain table; but may be done by the eye without an instrument; and being thus taken by guess in a rough draught, they will be exact enough, being only to look at; except it be such as are at any remarkable places, as at the

ends of hedges, at stiles, gates, &c. and these must be measured, or taken with the plain table. But all this need not be done till the draught is finished. And observe in all the hedges, what side the gutter or ditch is on, and to whom the fences belong.

8. When you have long stations, you ought to have a good instrument to take angles with, and the plain table may very properly be made use of, to take the several small internal parts, and such as cannot be taken from the main stations : as it is a very quick and ready instrument.

The New Method of Surveying.

Instead of the foregoing method, an ingenious friend (Mr. Abraham Crocker), after mentioning the new and improved method of keeping the field book by writing from bottom to top of the pages, observes that “ In the former method of measuring a large estate, the accuracy of it depends on the correctness of the instruments used in taking the angles. To avoid the errors incident to such a multitude of angles, other methods have of late years been used by some few skilful surveyors : the most practical, expeditious, and correct, seems to be the following.

“ As was advised in the foregoing method, so in this, choose two or more eminences, as grand stations, and measure a principal base line from one station to the other, noting every hedge, brook, or other remarkable object as you pass by it ; measuring also such short perpendicular lines to such bends of hedges as may be near at hand. From the extremities of this base line, or from any convenient parts of the same, go off with other lines to some remarkable object situated towards the sides of the estate, without regarding the angles they make with the base line or with one another ; still remembering to note every hedge, brook, or other object that you pass by. These lines, when laid down by intersections, will with the base line form a
grand

grand triangle upon the estate ; several of which, if need be, being thus laid down, you may proceed to form other smaller triangles and trapezoids on the sides of the former : and so on, until you finish with the enclosures individually.

“ This grand triangle being completed, and laid down on the rough plan paper, the parts, exterior as well as interior, are to be completed by smaller triangles and trapezoids.

“ When the whole plan is laid down on paper, the contents of each field might be calculated by the methods laid down below, at prob. 2. chap. 3.

“ In countries where the lands are enclosed with high hedges, and where many lanes pass through an estate, a theodolite may be used to advantage, in measuring the angles of such lands ; by which means, a kind of skeleton of the estate may be obtained, and the lane lines serve as the bases of such triangles and trapezoids as are necessary to fill up the interior parts.”

The method of measuring the other cross lines, offsets, and interior parts and enclosures, appears in the plan, fig. 1, pl. 28, Dictionary,

Another ingenious correspondent (Mr. John Rodham, of Richmond, Yorkshire,) has also communicated the following example of the new method of surveying, accompanied by the field-book, and its corresponding plan. His account of the method is as follows.

“ The field-book is ruled into three columns. In the middle one are set down the distances on the chain line at which any mark, offset, or other observation is made ; and in the right and left hand columns are entered the offsets and observations made on the right and left hand respectively of the chain line.

“ It is of great advantage, both for brevity and perspicuity, to begin at the bottom of the leaf and write upwards ; denoting the crossing of fences, by lines drawn across the middle column, or only a part of such a line on the right and left opposite the figures, to avoid confusion ; and the corners of fields, and other remark-
able

able turns in the fences where offsets are taken to, by lines joining in the manner the fences do, as will be best seen by comparing the book with the plan annexed to the field-book, in p. 212.

“ The marks called, *a, b, c, &c.* are best made in the fields, by making a small hole with a spade, and a chip or small bit of wood, with the particular letter upon it, may be put in, to prevent one mark being taken for another, on any return to it. But in general, the name of a mark is very easily had by referring in the book to the line it was made in. After the small alphabet is gone through, the capitals may be next, the print letters afterwards, and so on, which answer the purpose of so many different letters; or the marks may be numbered.

“ The letter in the left hand corner at the beginning of every line, is the mark or place measured *from*; and, that at the right hand corner at the end, is the mark measured *to*: But when it is not convenient to go exactly from a mark, the place measured from, is described *such a distance from one mark towards another*; and where a mark is not measured to, the exact place is ascertained by saying, *turn to the right or left hand, such a distance to such a mark*, it being always understood that those distances are taken in the chain line.

“ The characters used, are \lceil for *turn to the right hand*, \rceil for *turn to the left hand*, and \wedge placed over an offset, to shew that it is not taken at right angles with the chain line, but in the line with some straight fence; being chiefly used when crossing their directions, and it is a better way of obtaining their true places than by offsets at right angles.

“ When a line is measured whose position is determined, either by former work (as in the case of producing a given line, or measuring from one known place or mark to another) or by itself (as in the third side of a triangle) it is called a *fast line*, and a double line across the book is drawn at the conclusion of it; but if its position is not determined (as in the second side of a triangle) it is called

a *loose*

a *loose line*, and a single line is drawn across the book. When a line becomes determined in position, and is afterwards continued, a double line-half through the book is drawn.

“ When a loose line is measured, it becomes absolutely necessary to measure some line that will determine its position. Thus, the first line *ab*, being the base of a triangle, is always determined; but the position of the second side *bj*, does not become determined, till the third side *jb* is measured; then the triangle may be constructed, and the position of both is determined.

“ At the beginning of a line, to fix a loose line to the mark or place measured from, the sign of turning to the right or left hand must be added (as at *j* in the third line); otherwise a stranger, when laying down the work, may as easily construct the triangle *bjb* on the wrong side of the line *ab*, as on the right one: but this error cannot be fallen into, if the sign above named be carefully observed.

“ In choosing a line to fix a loose one, care must be taken that it does not make a very acute or obtuse angle; as in the triangle *pBr*, by the angle at *B* being very obtuse, a small deviation from truth, even the breadth of a point at *p* or *r*, would make the error at *B*, when constructed, very considerable; but by constructing the triangle *pBq*, such a deviation is of no consequence.

“ Where the words *leave off* are written in the field-book, it is to signify that the taking of offsets is from the ce discontinued; and of course something is wanting between that and the next offset.”

The field-book for this method, and the plan drawn from it, are contained in the four following pages, engraved on copper-plates.

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PROBLEM XII.

To survey a County, or Large Tract of Land.

1. Choose two, three, or four eminent places for stations; such as the tops of high hills or mountains, towers, or church steeples, which may be seen from one another; and from which most of the towns, and other places of note, may also be seen. And let them be as far distant from one another as possible. Upon these places raise beacons, or long poles, with flags of different colours flying at them: so as to be visible from all the other stations.

2. At all the places, which you would set down in the map, plant long poles with flags at them of several colours, to distinguish the places from one another; fixing them upon the tops of church steeples, or the tops of houses, or in the centres of lesser towns.

But you need not have these marks at many places at once, as suppose half a score at a time. For when the angles have been taken, at the two stations, to all these places, the marks may be moved to new ones; and so successively to all the places you want. These marks then being set up at a convenient number of places, and such as may be seen from both stations; go to one of these stations, and, with an instrument to take angles, standing at that station, take all the angles between the other station, and each of these marks, observing which is blue, which is red, &c. and which hand they lie on; and set all down with their colours. Then go to the other station, and take all the angles between the first station, and each of the former marks, and set them down with the others, each against his fellow with the same colour. You may, if you can, also take the angles at some third station, which may serve to prove the work, if the three lines intersect

perfect in that point where any mark stands. The marks must stand till the observations are finished at both stations; and then they must be taken down, and set up at fresh places. The same operations must be performed, at both stations, for these fresh places; and the like for others. Your instrument for taking angles must be an exceeding good one, made on purpose with telescopic sights; and of a good length of radius. A circumferentor is reckoned a good instrument for this purpose.

3. And though it be not absolutely necessary to measure any distance, because a stationary line being laid down from any scale, all the other lines will be proportional to it; yet it is better to measure some of the lines, to ascertain the distances of places in miles; and to know how many geometrical miles there are in any length; and from thence to make a scale to measure any distance in miles. In measuring any distance, it will not be exact enough to go along the high roads; by reason of their turnings and windings, and hardly ever lying in a right line between the stations, which must cause infinite reductions, and create endless trouble to make it a right line; for which reason it can never be exact. But a better way is to measure in a right line with a chain, between station and station, over hills and dales or level fields, and all obstacles. Only in case of water, woods, towns, rocks, banks, &c. where one cannot pass; such parts of the line must be measured by the methods of inaccessible distances; and besides, allowing for ascents and descents, when we meet with them. And a good compass that shews the bearing of the two stations, will always direct you to go straight, when you do not see the two stations; and in your progress, if you can go straight, you may take offsets to any remarkable places, likewise note the intersection of your stationary line with all roads, rivers, &c.

4. And from all the stations, and in the whole progress, be very particular in observing sea-coasts, river-mouths, towns, castles, houses, churches, windmills, watermills, trees, rocks, sands, roads, bridges, fords, ferries,

ferries, woods, hills, mountains, rills, brooks, parks, beacons, sluices, floodgates, locks, &c. and in general all things that are remarkable.

5. After you have done with your first and main station lines, which command the whole county; you must then take inner stations, at some places already determined; which will divide the whole into several partitions; and from these stations you must determine the places of as many of the remaining towns as you can. And if any remain in that part, you must take more stations, at some places already determined; from which you may determine the rest. And thus we must go through all the parts of the county, taking station after station, till we have determined all we want. And in general the station distances must always pass through such remarkable points as have been determined before, by the former stations.

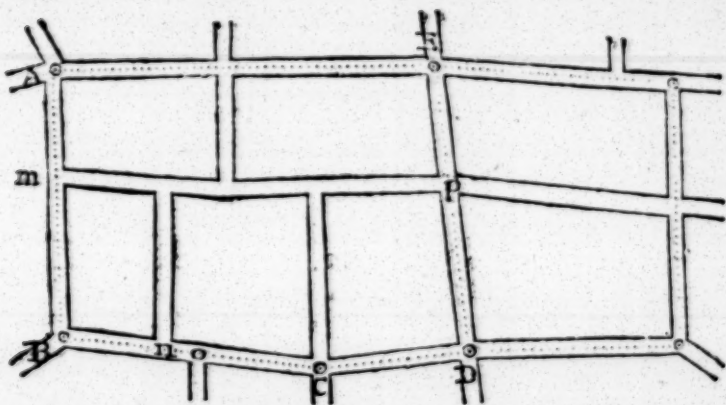
6. Lastly, the position of the station line you measure, or the point of the compass it lies on, must be determined by astronomical observation. Hang up a thread and plummet in the sun, over some part of the station line, and observe when the shadow runs along that line, and at that moment take the sun's altitude; then having his declination, and the latitude, the azimuth will be found by spherical trigonometry. The azimuth being the angle the station line makes with the meridian, therefore a meridian may easily be drawn through the map. Or a meridian may be drawn through it, by hanging up two threads in a line with the pole star, when it is just north, which may be known from astronomical tables. Or thus; observe the star Alioth, or that in the rump of the great bear, being that next the square, by a line and plummet when that star and the pole star come into a perpendicular; for at that time they are due north. Therefore two perpendicular lines being fixed at that moment, towards these two stars, will give the position of the meridian.

PROBLEM XIII.

To survey a Town or City.

This may be done with any of the instruments for taking angles, but best of all with the plain table, where every minute part is drawn while in sight. It is best also to have a chain of 50 feet long, divided into 50 links, and an offset-staff of 10 feet long.

Begin at the meeting of two or more of the principal streets, through which you can have the longest prospects, to get the longest station lines. There having fixed the instrument, draw lines of direction along those streets, using two men as marks, or poles set in wooden pedestals, or perhaps some remarkable places in the houses at the further ends, as windows, doors, corners, &c. Measure these lines with the chain, taking offsets with the staff, at all corners of streets, bendings, or windings, and to all remarkable things, as churches, markets, halls, colleges, eminent houses, &c. Then remove the instrument to another station, along one of these lines; and there repeat the same process as before. And so on till the whole is finished.



Thus, fix the instrument at A, and draw lines in the direction of all the streets meeting there; and measure AB,

AB, noting the street on the left at m. At the second station B, draw the directions of the streets meeting there; measure from B to C, noting the places of the streets at n and o as you pass by them. At the 3d station C, take the direction of all the streets meeting there, and measure CD. At D do the same, and measure DE, noting the place of the cross streets at p. And in this manner go through all the principal streets. This done, proceed to the smaller and intermediate streets; and lastly to the lanes, alleys, courts, yards, and every part which it may be thought proper to represent in the plan.

CHAPTER III.

OF PLANNING, COMPUTING, AND DIVIDING.

PROBLEM I.

To lay down the Plan of any Survey.

IF the survey was taken with a plain table, you have a rough plan of it already on the paper which covered the table. But if the survey was with any other instrument, a plan of it is to be drawn from the measures that were taken in the survey, and first of all a rough plan on paper.

To do this, you must have a set of proper instruments, for laying down both lines and angles, &c. as scales of various sizes, the more of them, and the more accurate, the better; scales of chords, protractors, perpendicular and parallel rulers, &c. Diagonal scales are best for the lines, because they extend to three figures, or chains and links, which are hundredth parts of chains. But in using the diagonal scale, a pair of compasses must be employed to take off the lengths of the principal lines very accurately.

accurately. But a scale with a thin edge divided, is much readier for laying down the perpendicular offsets to crooked hedges, and for marking the places of those offsets on the station line; which is done at only one application of the edge of the scale to that line, and then pricking off all at once the distances along it. Angles are to be laid down, either with a good scale of chords, which is perhaps the most accurate way; or with a large protractor, which is much readier when many angles are to be laid down at one point, as they are pricked off all at once round the edge of the protractor.

Very particular directions for laying down all sorts of figures cannot be necessary in this place, to any person who has learned practical geometry, and the construction of figures, with the use of his instruments. It may therefore be sufficient to observe, that all lines and angles must be laid down on the plan in the same order in which they were measured in the field, and in which they are written in the field-book; laying down first the angles for the position of lines, then the lengths of the lines, with the places of the offsets, and then the lengths of the offsets themselves, all with dry or obscure lines; then a black line drawn through the extremities of all the offsets, will be the hedge or bounding line of the field, &c. After the principal bounds and lines are laid down, and made to fit or close properly, proceed next to the smaller objects, till you have entered every thing that ought to appear in the plan, as houses, brooks, trees, hills, gates, stiles, roads, lanes, mills, bridges, woodlands, &c. &c.

The north side of a map or plan is commonly placed uppermost, and a meridian somewhere drawn, with the compass or flower-de-luce pointing north. Also, in a vacant place, a scale of equal parts or chains must be drawn, with the title of the map in conspicuous characters, and embellished with a compartment. All hills must be shadowed, to distinguish them in the map. Colour the hedges with different colours; represent hilly grounds by broken hills and valleys; draw single dotted lines for
foot-

foot-paths, and double ones for horse or carriage roads. Write the name of each field and remarkable place within it, and, if you choose, its content in acres, roods, and perches.

In a very large estate, or a county, draw vertical and horizontal lines through the map, denoting the spaces between them by letters placed at the top, and bottom, and sides, for readily finding any field or other object, mentioned in a table.

In mapping counties, and estates that have uneven grounds of hills and valleys, reduce all oblique lines, measured up hill and down hill, to horizontal straight lines, if that was not done during the survey, before they were entered in the field-book, by making a proper allowance to shorten them. For which purpose there is commonly a small table engraven on some of the instruments for surveying.

PROBLEM II.

To Compute the Contents of Fields.

1. Compute the contents of the figures, whether triangles, or trapeziums, &c. by the proper rules for the several figures laid down in measuring; multiplying the lengths by the breadths, both in links, and divide by 2; the quotient is acres, after you have cut off five figures on the right for decimals. Then bring these decimals to roods and perches, by multiplying first by 4, and then by 40. An example of which is given in the description of the chain, page 176.

2. In small and separate pieces, it is usual to compute their contents from the measures of the lines taken in surveying them, without making a correct plan of them.

Thus,

Thus, in the triangle in prob. iv. page 190, where we had $AP = 794$, and $AB = 1321$
 $PC = 826$

$$\begin{array}{r} 7926 \\ 2642 \\ \hline 10568 \end{array}$$

$$\begin{array}{r} 2) 10'91146 (\\ 5'45573 \quad \text{ac r p} \\ \hline 4 \text{ Anf. } 5 \quad 1 \quad 33 \end{array}$$

$$\begin{array}{r} 1'82292 \\ 40 \\ \hline \end{array}$$

$$\begin{array}{r} 32'91680 \\ \hline \end{array}$$

Or the first example to prob. v. page 191, thus :

$$\begin{array}{r|l} \text{AE } 214 & 210 \text{ ED} \\ \text{AF } 362 & 306 \text{ FB} \\ \text{AC } 592 & \hline & 516 \text{ sum of perp.} \\ & 592 \text{ AC} \end{array}$$

$$\begin{array}{r} 1032 \\ 4644 \\ \hline 2580 \end{array}$$

$$\begin{array}{r} 2) 3'05472 \\ 1'52736 \quad \text{ac r p} \\ \hline 4 \text{ Anf. } 1 \quad 2 \quad 4 \end{array}$$

$$\begin{array}{r} 2'10944 \\ 40 \\ \hline \end{array}$$

$$\begin{array}{r} 4'37760 \\ \hline \end{array}$$

Or

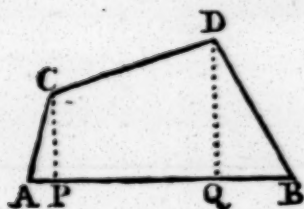
Or the 2d example to the same prob. v. thus:

AP	110	352	PC
AQ	745	595	QD
AB	1110		
PC	352	PC	352
AP	110	QD	595
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2 APC	38720	sum	947
		PQ	635
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			4735
			2841
			5682
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2 PCDQ	601345		

QD	595
QB	365
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	2975
	3570
	1785
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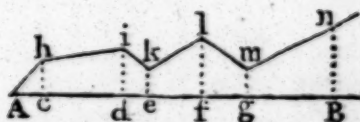
217175	= 2 QDB
601345	= 2 PCDQ
38720	= 2 APC

2) 8.57240	= dou. the whole
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3. In pieces bounded by very crooked and winding hedges, measured by offsets, all the parts between the offsets are most accurately measured separately as small trapezoids. Thus, for the example to prob. III. p. 189, where

AC	45	62	ch
Ad	220	84	di
Ae	340	70	ek
Af	510	88	fl
Ag	634	57	gm
AB	785	91	Bn
<hr/>		<hr/>	



I.

Then

						Then					
Ac 45	ch 62	di 84	ek 70	fl 88	gm 57	ch 62	di 84	ek 70	fl 98	gm 57	gn 91
—	—	—	—	—	—	—	—	—	—	—	—
90	146	154	168	145	148						
270	cd 175	de 120	ef 170	fg 124	gB 151						
—	—	—	—	—	—						
2790	730	18480	11760	580	148						
—	1022	—	168	290	740						
	146			145	148						
	—		28560	—	—						
	25550		—	17980	22348						

2790

25550

18480

28560

17980

22348

2) 115708

57854

4

231416

40

1256640

ac r p

Content o 2 12

4. Sometimes such pieces as that above, are computed by finding a mean breadth, by dividing the sum of the offsets by the number of them, accounting that for one of them where the boundary meets the station line, as at A; then multiply the length AB by that mean breadth.

Thus:

Thus:

00	785	AB	
62	66	mean breadth	
84	—		
70	4710	ac r p	
98	4710	Content	0 2 2 by this method,
57	—		which is 10 perches too little.
91	51810		
—	4		
7) 462	—		
66	207240	But this method is always erroneous,	
—	40	except when the offsets stand at equal	
	—	distances from one another.	
	289600		
	—		

5. But in larger pieces, and whole estates, consisting of many fields, it is the common practice to make a rough plan of the whole, and from it compute the contents quite independent of the measures of the lines and angles that were taken in surveying. For then new lines are drawn in the fields in the plan, so as to divide them into trapeziums and triangles, the bases and perpendiculars of which are measured on the plan by means of the scale from which it was drawn, and so multiplied together for the contents. In this way, the work is very expeditiously done, and sufficiently correct; for such dimensions are taken, as afford the most easy method of calculation; and, among a number of parts, thus taken and applied to a scale, it is likely that some of the parts will be taken a small matter too little, and others too great; so that they will, upon the whole, in all probability, very nearly balance one another. After all the fields, and particular parts, are thus computed separately, and added all together into one sum; calculate the whole estate independent of the fields, by dividing it into large and arbitrary triangles and trapeziums, and add these also together. Then if this sum be equal to the former, or nearly so, the work is right; but if the sums have any considerable difference, it is wrong,

and they must be examined and recomputed till they nearly agree.

A specimen of dividing into one triangle, or one trapezium, which will do for most single fields, may be seen in the examples to the last problem; and a specimen of dividing a large tract into several such trapeziums and triangles, in prob. vi. of chapter 11 of Surveying, page 193, where a piece is so divided, and its dimensions taken and set down; and again at prob. vi. of Mensuration of Surfaces, where the contents of the same piece are computed.

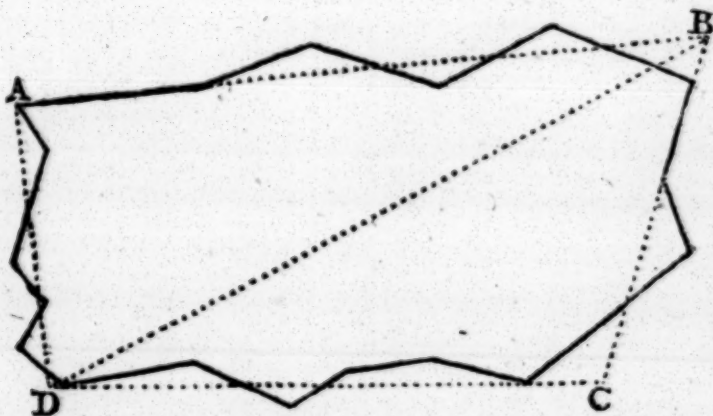
6. But the chief secret in computing, consists in finding the contents of pieces bounded by curved or very irregular lines, or in reducing such crooked sides of fields or boundaries to straight lines, that shall inclose the same or equal area with those crooked sides, and so obtain the area of the curved figure by means of the right-lined one which will commonly be a trapezium. Now this reducing the crooked sides to straight ones, is very easily and accurately performed, thus: Apply the straight edge of a thin clear piece of lanthorn-horn to the crooked line, which is to be reduced in such a manner, that the small parts cut off from the crooked figure by it, may be equal to those which are taken in; which equality of the parts included and excluded you will presently be able to judge of very nicely by a little practice: then with a pencil, or point of a tracer, draw a line by the straight edge of the horn. Do the same by the other sides of the field or figure. So shall you have a straight sided figure equal to the curved one; the content of which, being computed as before directed, will be the content of the curved figure proposed.

Or, instead of the straight edge of the horn, a horse-hair may be applied across the crooked sides in the same manner; and the easiest way of using the hair, is to string a small slender bow with it, either of wire, or cane, or whale-bone, or suchlike slender elastic matter; for, the bow keeping it always stretched, it can be easily and neatly applied with one hand, while the other is at liberty

to make two marks by the side of it, to draw the straight line by.

EXAMPLES.

Thus, let it be required to find the contents of the same figure as in prob. ix of the last chapter, page 198, to a scale of 4 chains to an inch.



Draw the four dotted straight lines AB, BC, CD, DA, cutting off equal quantities on both sides of them, which they do as near as the eye can judge: so is the crooked figure reduced to an equivalent right lined one of four sides ABCD. Then draw the diagonal BD, which, by applying a proper scale to it, measures 1256. Also the perpendicular or nearest distance, from A to this diagonal, measures 456; and the distance of C from it, is 428.

$$\begin{array}{r}
 \text{Then} \quad 456 \\
 \quad \quad 428 \\
 \hline
 \quad \quad 884 \\
 \quad \quad 1256 \\
 \hline
 \quad \quad 5024 \\
 \quad 10048 \\
 10048 \\
 \hline
 2) 1110304 \\
 \quad 555152 \\
 \quad \quad 4 \\
 \hline
 \quad 220608 \\
 \quad \quad 40 \\
 \hline
 \quad 824320 \quad \text{ac r p} \\
 \hline
 \quad \quad \text{Content } 5 \quad 2 \quad 8
 \end{array}$$

And thus the content of the trapezium, and consequently of the irregular figure, to which it is equal, is easily found to be 5 acres, 2 roods, 8 perches.

PROBLEM III.

To transfer a Plan to another Paper, &c.

After the rough plan is completed, and a fair one is wanted; this may be done, either on paper or vellum, by any of the following methods.

FIRST METHOD.

Lay the rough plan over the clean paper, and keep them always pressed flat and close together, by weights laid on them. Then, with the point of a fine pin or pricker, prick through all the corners of the plan to be copied. Take them asunder, and connect the pricked points, on the clean paper, with lines; and it is done.

This

This method is only to be practised in plans of such figures as are small and tolerably regular, or bounded by right lines.

SECOND METHOD.

Rub the back of the rough plan over with black lead powder; and lay the said black part on the clean paper on which the plan is to be copied, and in the proper position. Then with the blunt point of some hard substance, as brass or such like, trace over the lines of the whole plan; pressing the tracer so much as that the black lead under the lines may be transferred to the clean paper: after which take off the rough plan, and trace over the leaden marks with common ink, or with Indian ink, &c.—Or, instead of blacking the rough plan, you may keep constantly a blacked paper to lay between the plans.

THIRD METHOD.

Another method of copying plans, is by means of squares. This is performed by dividing both ends and sides of the plan which is to be copied, into any convenient number of equal parts, and connecting the corresponding points of division with lines; which will divide the plan into a number of small squares. Then divide the paper, on which the plan is to be copied, into the same number of squares, each equal to the former when the plan is to be copied of the same size, but greater or less than the others, in the proportion in which the plan is to be increased or diminished, when of a different size. Lastly, copy into the clean squares the parts contained in the corresponding squares of the old plan; and you will have the copy, either of the same size, or greater or less in any proportion.

FOURTH METHOD.

A fourth method is by the instrument called a pentagraph, which also copies the plan in any size required.

FIFTH METHOD.

But the neatest method of any is this. Procure a copying frame or glass, made in this manner; namely, a large square of the best window glass, set in a broad frame of wood, which can be raised up to any angle, when the lower side of it rests on a table. Set this frame up to any angle before you, facing a strong light; fix the old plan and clean paper together with several pins quite around, to keep them together, the clean paper being laid uppermost, and over the face of the plan to be copied. Lay them with the back of the old plan, over the glass, namely, that part which you intend to begin at to copy first; and, by means of the light shining through the papers, you will very distinctly perceive every line of the plan through the clean paper. In this state then trace all the lines on the paper with a pencil. Having drawn that part which covers the glass, slide another part over the glass, and copy it in the same manner: then another part. And so on, till the whole be copied.

Then, take them asunder, and trace all the pencil-lines over with a fine pen and Indian ink, or with common ink.

And thus you may copy the finest plan, without injuring it in the least.

When the lines, &c. are copied on the clean paper or vellum, the next business is to write in such names, remarks, or explanations as may be judged necessary; laying down the scale for taking the lengths of any parts, a flower-de-luce to point out the direction, and the proper title, ornamented with a compartment; and illustrating or colouring every part, in such manner as shall seem most natural, such as shading rivers or brooks with crooked lines, drawing the representations of trees, bushes, hills, woods, hedges, houses, gates, roads, &c. in their proper places; running a single dotted line for a foot path, and a double one for a carriage road; and either representing the bases or the elevations of buildings, &c.

CONIC

CONIC SECTIONS

AND THEIR SOLIDS.

DEFINITIONS.

1. **C**ONIC Sections are the plane figures formed by cutting a cone.

According to the different positions of the cutting plane there will arise five different figures or sections.

2. If the cutting plane pass through the vertex, and any part of the base, the section will be a triangle



3. If the cone be cut parallel to the base, the section will be a circle.



4. The section is called an ellipsis, when the cone is cut obliquely through both sides.



5. The section is a parabola, when the cone is cut parallel to one of its sides.

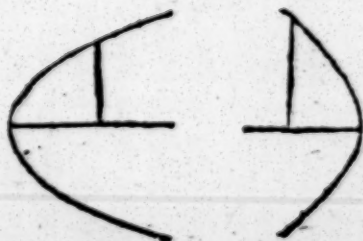


6. The section is an hyperbola, when the cutting plane meets the opposite cone continued above the vertex, where it will make another section or hyperbola.



7. The vertices of any section, are the points where the cutting plane meets the opposite sides of the cone.

8. The transverse axis, is the line between the two vertices. And the middle point of the transverse is the centre of the conic section.



9. The conjugate axis, is a line drawn through the centre, and perpendicular to the transverse.

10. An ordinate, is a line perpendicular to the axis.

11. An absciss, is a part of the axis between the ordinate and the vertex.

12. A spheroid, or ellipsoid, is a solid generated by the revolution of an ellipse about one of its axis. It is a prolate one, when the revolution is made about the transverse axis; and oblate, when about the conjugate.



13. A conoid is a solid formed by the revolution of a parabola, or hyperbola, about the axis. And is accordingly called parabolic, or hyperbolic.—The parabolic conoid is also called a paraboloid; and the hyperbolic conoid, an hyperboloid.



14. A spindle is formed by any of the three sections revolving about a double ordinate, like the circular spindle.

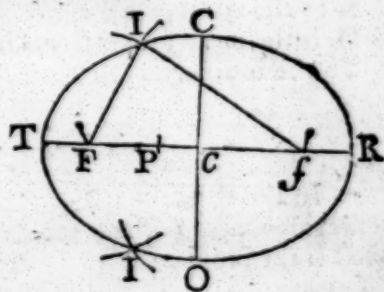
15. A segment, of any of these figures, is a part cut off at the top, by a plain parallel to the base.

16. And a frustum is the part left next the base, after the segment is cut off.

PROBLEM I.

To describe an Ellipse.

Let TR be the transverse, CO the conjugate, and c the centre. With the radius Tc and centre C, describe an arc cutting TR in the points F, f ; which are called the two foci of the ellipse.



L 6

Assume

Assume any point P in the transverse; then with the radii PT , PR , and centres F , f , describe two arcs intersecting in I ; which will be a point in the curve of the ellipse.

And thus, by assuming a number of points P in the transverse, there will be found as many points in the curve as you please. Then, with a steady hand, draw the curve through all these points.

Otherwise, with a Thread.

Take a thread of the length of the transverse TR , and fasten its ends with two pins in the foci F , f . Then stretch the thread, and it will reach to I in the curve: and by moving a pencil round, within the thread, keeping it always stretched, it will trace out the ellipse.

PROBLEM II.

In an Ellipse, to find the Transverse, or Conjugate, or Ordinate, or Absciss: having the other three given.

CASE I.

To find the Ordinate.

As the transverse :
Is to the conjugate ::
So is the mean proportional between the two abscisses :
To the ordinate.

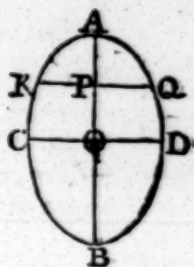
EXAMPLES.

1. In the ellipse $ADEC$, the transverse AB is 70, the conjugate CD is 50, and the abscisses AP 14, and PB 56; what is the ordinate PQ ?

First

First

$$\begin{array}{r}
 56 = PB \\
 14 = AP \\
 \hline
 224 \\
 56 \\
 \hline
 784 \text{ (28 mean} \\
 4 \text{ between 56} \\
 \text{and 14)} \\
 \hline
 48 \overline{) 384} \\
 \underline{384} \\
 0
 \end{array}$$



Then $70 : 50 :: 28 : 20 = PQ$ the ordinate.

Ex. 2. If the transverse be 80, the conjugate 60, and an absciss 16; required the ordinate? Ans. 24.

CASE II.

To find the Absciss.

From the square of half the conjugate, take the square of the ordinate; and extract the square root of the remainder. Then say,

As the conjugate	:
Is to the transverse	::
So is that square root	:
To half the difference of the abscisses.	

Then add this half difference to half the transverse, for the greater absciss; and subtract it, for the less absciss.

EXAMPLES.

1. The transverse AB is 70, the conjugate CD is 50 and the ordinate PQ is 20; required the abscisses AP and PB?

First

First

$$\begin{array}{r} 25 \\ 25 \\ \hline \end{array}$$

$$\begin{array}{r} 125 \\ 50 \\ \hline \end{array}$$

$$625 = CO^2$$

$$400 = PQ^2$$

$$\begin{array}{r} 225 \text{ (} 15 \\ 1 \\ \hline \end{array}$$

$$25 \overline{) \begin{array}{r} 125 \\ 125 \\ \hline \end{array}}$$

Then

$$\text{As } 50 : 70 :: 15 : 21 \text{ half dif.}$$

$$35 \text{ half AB}$$

$$56 = PB$$

$$14 = AP$$

Ex. 2. What are the two abscisses to the ordinate 24, the axes being 80 and 60? Ans. 16 and 64.

CASE III.

To find the Conjugate.

As the mean proportional between the abscisses :
 Is to the ordinate ::
 So is the transverse :
 To the conjugate.

Note. In the same manner, the transverse may be found from the conjugate; using here the abscisses of the conjugate, and their ordinate perpendicular to the conjugate.

EXAMPLES.

1. The transverse being 180, the ordinate 16; and the greater absciss 144; required the conjugate?

180 trans-

180 transverse
 144 greater abs.
 36 less abs.

864
 432

5184 (72 : 16 :: 180 : 40 the conjugate.
 49 16

142 | 284
 | 284

1080
 18

72) 2880 (40
 288

Ex. 2. The transverse being 70, the ordinate 20, and
 absciss 14; what is the conjugate? Ans. 50.

CASE IV.

To find the Transverse.

From the square of half the conjugate, subtract the square of the ordinate; and extract the root of the remainder. Next add this root to the half conjugate, if the less absciss be given, but subtract it when the greater absciss is given, reserving the sum or difference.

Then say,

As the square of the ordinate :
 Is to the rectangle of the absciss and conjugate ::
 So is the reserved sum or difference :
 To the transverse.

EXAMPLES.

1. If the conjugate be 50, the ordinate 20, and the less absciss 14; what is the transverse?

First

CONIC SECTIONS

First	Then	
25	20	14
25	20	50
<hr/>		
125	400	700 :: 40 : 70 the transf.
50	<hr/>	<hr/>
<hr/>		
625		
400		
<hr/>		
225	(15	
1	25	
<hr/>		
25	125	40
	125	—
<hr/>		

Ex. 2. The conjugate being 40, the ordinate 16, and the less absciss 36; required the transverse? Ans. 180.

PROBLEM III.

To find the Circumference of an Ellipse.

Add the two axes together, and multiply the sum by 1.5708, for the circumference, nearly.

EXAMPLES.

1. Required the circumference of the ellipse whose two axes are 70 and 50?

$$\begin{array}{r}
 70 \\
 50 \\
 \hline
 120 \text{ sum.} \\
 1.5708 \\
 \hline
 188.4960 \text{ circumf. nearly.} \\
 \hline
 \end{array}$$

Ex. 2.

Ex. 2. What is the periphery of an ellipse whose two axes are 24 and 20?
 Anf. 69.1152.

PROBLEM IV.

To find the Area of an Ellipse.

Multiply the transverse by the conjugate; then that product multiplied by .7854, will be the area.

Or, multiply .7854 first by the one axis, and the product again by the other.

EXAMPLES.

1. To find the area of the ellipse whose two axes are 70 and 50.

$$\begin{array}{r}
 .7854 \\
 50 \\
 \hline
 39.2700 \\
 70 \\
 \hline
 2748.9000 \text{ anf.} \\
 \hline
 \end{array}$$

Ex. 2. What is the area of the ellipse whose two axes are 24 and 18?
 Anf. 339.2928.

PROBLEM V.

To find the Area of an Elliptic Segment.

Divide the height of the segment by that axis of the ellipse of which it is a part; and find, in the table of circular segments at the end of the book, a circular segment having the same versed sine as this quotient. Then multiply continually together, this segment, and the two axes, for the area required.

EX-

EXAMPLES.

1. What is the area of an elliptic segment RAQ, whose height AP is 20; the transverse AB being 70, and the conjugate CD 50?

70) 20 ($\cdot 285\frac{1}{2}$ the tab. vers.

The correspond. seg.

is $\cdot 185166$

70

12.961620

50

648.081000



Ex. 2. What is the area of an elliptic segment, cut off parallel to the shorter axis, the height being 10, and the axes 25 and 35; Ans. 162.0210 .

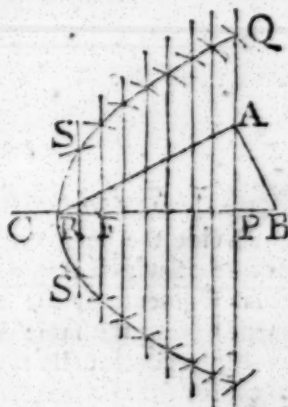
Ex. 3. What is the area of the elliptic segment, cut off parallel to the longer axis, the height being 5, and the axes 25 and 35? Ans. 97.8458 .

PROBLEM VI.

To describe or construct a Parabola.

VP being an absciss, and PQ its given ordinate; bisect PQ in A, join AV, and draw AP perpendicular to it; then transfer PB to VF and VC in the axis produced. So shall F be what is called the focus.

Draw several double ordinates SRS, &c. Then with the radii CR, &c. and the centre F, describe arcs cutting the corresponding ordinates in the points S, &c.



Then.

Then draw the curve through all the points S, &c.

PROBLEM VII.

To find any Parabolic Abscifs or Ordinate.

The abscisses are to each other as the squares of their ordinates; that is,

As any abscifs is to the square of its ordinate,

So is any other abscifs, to the square of its ordinate.

Or as the square root of any abscifs, is to its ordinate,

So is the square root of another abscifs, to its ordinate.

EXAMPLES.

1. The abscifs VB is 9, and its ordinate AB is 6; required the ordinate DE whose abscifs VE is 16.

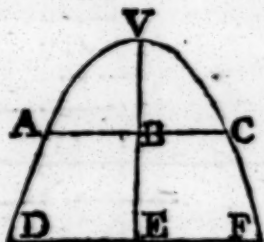
Here $\sqrt{9}$ is 3, and $\sqrt{16}$ is 4.

Then $3 : 6 :: 4 : 8 = DE$ required.

Or if the ordinate DE were given = 8, to find its abscifs VE.

Then $6^2 = 36$, and $8^2 = 64$.

Hence $36 : 64 :: 9 : 16 = VE$ required.



Ex. 2. If an abscifs be 8, and its ordinate 10; required the ordinate whose abscifs is 18? Anf. 15.

Ex. 3. If an abscifs be 18, and its ordinate 15; what is the abscifs whose ordinate is 10? Anf. 8.

PROBLEM VIII.

To find the Length of a Parabolic Curve.

To the square of the ordinate, add $\frac{4}{3}$ of the square of the abscifs; extract the square root of the sum, and double it for the length of the curve, cut off by the double ordinate, nearly.

EX-

EXAMPLES.

1. The absciss VB being 2, and the ordinate AB 6; required the length of the curve AVC?

$$2 = VB$$

$$2$$

$$\hline$$

$$4 = VB^2$$

$$4$$

$$\hline$$

$$3 \overline{) 16}$$

$$\hline$$

$$5.3333$$

$$36$$

$$\hline$$

$$41.3333 \text{ (} 6.4291 \text{ root}$$

$$36$$

$$\hline$$

$$124 \overline{) 533}$$

$$4 \overline{) 496}$$

$$\hline$$

$$1282 \overline{) 3733}$$

$$2 \overline{) 2564}$$

$$\hline$$

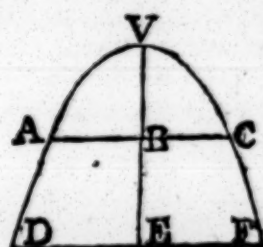
$$1284 \overline{) 1169} \text{ (91}$$

$$1156$$

$$\hline$$

$$13$$

$$\hline$$



$$12.8582 = \text{arc AVC nearly.}$$

Ex. 2, What is the length of the parabolic curve, whose absciss is 3, and ordinate 8? Ans. 17.4356

PROBLEM IX.

To find the Area of a Parabola.

Multiply the base by the height, and take $\frac{2}{3}$ of the product for the area.

EX-

EXAMPLES.

1. Required the area of the parabola AVCA, the absciss VB being 2, and the ordinate AB 6?

$$\begin{array}{r}
 12 \\
 2 \\
 \hline
 24 \\
 2 \\
 \hline
 3 \) \ 48 \\
 \underline{16} \text{ anf.}
 \end{array}$$

Ex. 2. What is the area of a parabola whose absciss is 10, and ordinate 8? Anf. $106\frac{2}{3}$.

PROBLEM X.

To find the Area of a Parabolic Frustum

Cube each end of the frustum, and subtract the one cube from the other; then multiply that difference by double the altitude, and divide the product by triple the difference of their squares, for the area.

EXAMPLES.

1. Required the area of the parabolic frustum ACFD, AC being 6, DF 10, and the altitude BE 4.

Ends	Sqrs.	Cubes
10	100	1000
6	36	216
—	64	784
	3	8
192)	6272 ($32\frac{1}{3} = 32\frac{2}{3}$ anf.
		576
		512
		384
		128

Ex. 2,

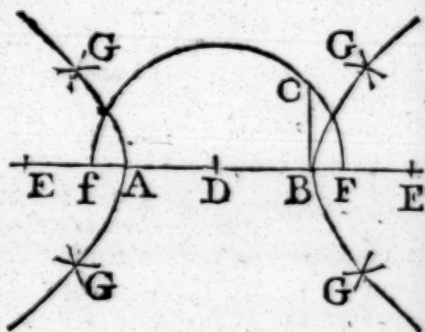
Ex. 2. What is the area of the parabolic frustum, whose two ends are 6 and 10, and its altitude 3? Ans. $24\frac{1}{2}$.

PROBLEM XI.

To construct or describe an Hyperbola.

Let D be the centre of the hyperbola, or the middle of the transverse AB; and BC perpendicular to AB, and equal to half the conjugate.

With centre D, and radius DC, describe an arc, meeting AB produced in F and f, which are the two focus points of the hyperbola.



Then, assuming several points E in the transverse produced, with the radii AE, BE, and centres f, F, describe arcs intersecting in the several points G; through all which points draw the hyperbolic curve.

PROBLEM XII.

In an Hyperbola to find the Transverse, or Conjugate, or Ordinate, or Absciss.

CASE I.

To find the Ordinate.

As the transverse :
Is to the conjugate :
So is the mean propor. between the two abscisses :
To the ordinate.

Note.

Note. In the hyperbola, the less absciss added to the axis, gives the greater absciss.

EXAMPLES.

1. If the transverse be 24, the conjugate 21, and the less absciss VB 8; what is the ordinate AB?

24 transf.
8 less abf.

—
32 greater abf.
8

—
256 (16 mean propor.
1

—
26 | 156
156
—



Then $24 : 21 :: 16 : 14 = AB$ required.

Ex. 2. The transverse being 60, the conjugate 36, and the less absciss 20, required the ordinate? *Ans.* 24.

CASE II.

To find the Absciss.

To the square of half the conjugate, add the square of the ordinate? and extract the square root of the sum. Then say,

As the conjugate	:
Is to the transverse	::
So is that square root	:
To half the sum of the abscisses.	

Then, to this sum, add half the transverse, for the greater absciss; and subtract it, for the less absciss.

EXAMPLES.

1. The transverse being 24, and the conjugate 21: required the two abscisses to the ordinate AB 14?

First $10.5 = \frac{1}{2}$ conj. 14 ord.

10.5	14
525	56
105	14
110.25	196
196	

306.25 (17.5 the square root. Then

27	206	As 21 : 24 :: 17.5 :	20 half sum
7	189		12 half trans.
345	1725	8	
5	1725	7) 140.0	32 greater abs.
		20	8 less absciss.

Ex. 2. The transverse being 60, the conjugate 36; required the two abscisses to the ordinate 24?

Anf. 80 and 20.

CASE III.

To find the Conjugate.

As the mean proportion between the abscisses :
 Is to the ordinate ::
 So is the transverse :
 To the conjugate.

E X.

EXAMPLES.

1. The transverse being 24, the less absciss VB 8, and its ordinate AB 14? what is the conjugate?

$$\begin{array}{r} \text{First} \quad 24 \\ - \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 256 \text{ (16 the mean. } \\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 26 \overline{) 156} \\ 6 \overline{) 156} \\ \hline \end{array}$$

Then

$$\text{As } 16 : 14 :: 24 : 21 \text{ Anf.}$$

$$\begin{array}{r} 7 \\ 8 \overline{) 168} \\ 21 \\ \hline \end{array}$$

Ex. 2. What is the conjugate to the hyperbola, whose transverse is 60, and ordinate 24, and the less absciss 20? Anf. 36.

CASE IV.

To find the Transverse.

To the square of half the conjugate add the square of the ordinate, and extract the square root of the sum.

Next, to this root add the half conjugate when the less absciss is used, but subtract it when the greater absciss is used; reserving the sum or difference. Then say,

As the square of the ordinate :
Is to the product of the absciss and conjugate ::
So is the reserved sum or difference :
To the transverse.

EXAMPLES.

1 The less absciss VB being 8, and its ordinate AB 14; required the transverse to the conjugate 21?

M

First

First	10.5	14
	10.5	14
	<hr/>	<hr/>
	525	56
	105	14
	<hr/>	<hr/>
	110.25	196
	196	<hr/>
	<hr/>	
	306.25	(17.5
	1	10.5
	<hr/>	<hr/>
27	206	28.0
7	189	<hr/>
	<hr/>	
345	1725	
5	1725	
	<hr/>	

Then

As 196 : 168 :: 28 :

Or 7 : 6 :: 28 : 24 Anf.

Ex. 2. What is the transverse of the hyperbola, whose conjugate is 36; the less absciss being 20, and its ordinate 24 ?

Anf. 60.

PROBLEM XIII.

To find the Length of an Hyperbolic Curve.

1. To 21 times the square of the conjugate, add 9 times the square of the transverse; and to the same 21 times the square of the conjugate, add 19 times the square of the transverse; and multiply each sum by the absciss.
2. To each of these two products add 15 times the product of the transverse and square of the conjugate.
3. Then as the less sum is to the greater, so is the double ordinate, to the length of the curve nearly.

EXAMPLES.

1. Required the length of the curve AVC to the absciss VB 20 and ordinate AB 24; the two axes being 60 and 36;

	36	27216	27216	1296
	36	32400	68400	60
	<hr/>	<hr/>	<hr/>	<hr/>
	216	59616	95616	77760
	108	20	20	15
	<hr/>	<hr/>	<hr/>	<hr/>
sq. conju.	1296	1192320	1912320	388800
	21	1166400	1166400	777600
	<hr/>	<hr/>	<hr/>	<hr/>
	1296	2358720	3078720	1166400
	2592	<hr/>	<hr/>	<hr/>
	<hr/>			
	27216			
	<hr/>			

Then $2358720 : 3078720 :: 48 : 62.6520$ the whole curve

	<hr/>
	2462976
	1231488
	<hr/>
2358772)	14777856 (62.6520 Anf.
.....	1415232
	<hr/>
	62553
	47174
	<hr/>
	15379
	14152
	<hr/>
	1227
	1179
	<hr/>
	48
	47
	<hr/>
	1
	<hr/>



Ex. 2. What is the length of the whole curve to the ordinate 10. the transverse and conjugate axes being 80 and 60? Ans. 20.601.

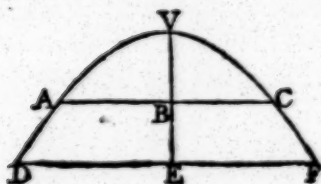
PROBLEM XIV.

To find the Area of an Hyperbola.

1. To $\frac{1}{2}$ of the absciss add the transverse: multiply the sum by the absciss; extract the square root of the product.
2. Multiply the transverse by the absciss, and extract the root of that product also.
3. To 21 times the first root, add 4 times the second root; multiply the sum by double the product of the conjugate and absciss; then divide by 75 times the transverse, for the area nearly.

EXAMPLES.

1. Required the area of the hyperbola AVCA, whose absciss VB is 10, the transverse and conjugate being 30 and 18?



$$\begin{array}{r}
 474^{\circ}0041586 \\
 \underline{720} \\
 94800831720 \\
 \underline{33180291102} \\
 3412829941920 \\
 \underline{4} \\
 9^{\circ}000 \quad) \quad 1365^{\circ}131^{\circ}9767680 \\
 \underline{151^{\circ}68133} \text{ area required.}
 \end{array}$$

Ex. 2. What is the area of the hyperbola to the abscissa 25, the two axes being 50 and 30?

Ans. $805^{\circ}090868$.

PROBLEM XV.

To find the Solidity of a Spheroid.

Square the revolving axis, multiply that square by the fixed axis, and multiply the product by $\cdot 5236$ for the content.

EXAMPLES.

1. Required the solidity of the prolate spheroid ACBD, whose axes are AB 50 and CD 30?

$$\begin{array}{r}
 30 \\
 \underline{30} \\
 900 \\
 \underline{50} \\
 45000
 \end{array}
 \begin{array}{r}
 \cdot 5236 \\
 45000 \\
 \underline{26180000} \\
 20944 \\
 \underline{23562^{\circ}0000} \text{ Ans.}
 \end{array}$$

Ex. 2.

Ex. 2. What is the content of an oblate spheroid, whose axes are 50 and 30? Ans. 39270.

Ex. 3. What is the solidity of a prolate spheroid, whose axes are 9 and 7? Ans. 230'9076.

PROBLEM XVI.

To find the Solidity of the Segment of a Spheroid.

CASE I.

When the Base is Circular, or Parallel to the revolving Axis.

From triple the fixed axe, take double the height of the segment; multiply that difference by the square of the height, and the product again by $\cdot 5236$.

Then as the square of the fixed axe is to the square of the revolving axe, so is the last product, to the content of the segment.

EXAMPLES.

1. Required the content of the segment of a prolate spheroid, the height AG being 5, the fixed axe AB 50, and the revolving axe CD 30?

150	$\cdot 5236$
10	3500
<hr/>	
140	2618000
25	15708
<hr/>	
700	1832'6000
28	
<hr/>	
3500	
<hr/>	

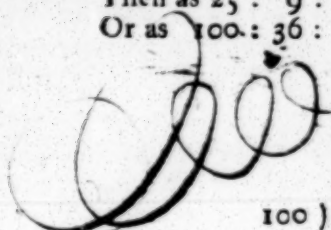


M 4

Then

Then as 25 : 9 :: 1832.6 :

Or as 100 : 36 :: 1832.6 : 659.736



36	
109956	
54978	

100) 65973.6 (659.736 Answer.

Ex. 2. If the axes of a prolate spheroid be 10 and 6, required the area of the segment whose height is 1, and its base parallel to the revolving axe? Ans. 5.277888.

Ex. 3. The axes of an oblate spheroid being 50 and 30, what is the content of the segment, the height being 6, and its base parallel to the revolving axe?

Ans. 4084.07.

CASE II.

When the Base is Perpendicular to the Revolving Axe.

From triple the revolving axe, take double the height of the segment; multiply that difference by the square of the height, and the product again by .5236.

Then as the revolving axe, is to the fixed axe,
So is the last product, to the content.

EXAMPLES.

1. In the prolate spheroid ACBD, the fixed axe AB is 50, the revolving axe CD 30; required the solidity of the segment CEF, its height CG being 6?

50

50

50

50

50

50

50

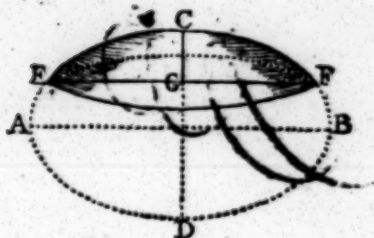
50

50

50

50

90	5236
12	2808
<hr/>	
78	41888
36	41888
<hr/>	
468	10472
234	14702688
<hr/>	
2808	
<hr/>	



Then as 30 : 50 :: 14702688 : 2450448.

$$\begin{array}{r}
 5 \\
 \hline
 3 \overline{) 73513440} \\
 \hline
 24504180 \text{ Answer.} \\
 \hline
 \end{array}$$

Ex. 2. In an oblate spheroid, whose axes are 50 and 30, required the content of the segment whose height is 5, its base being perpendicular to the revolving axe?

Anf. 109956.

PROBLEM XVII.

To find the Content of the Middle Frustum of a Spheroid.

CASE I.

When the Ends are Circular, or Parallel to the Revolving Axe.

To double the square of the middle diameter, add the square of the diameter of one end; multiply this sum by the length of the frustum, and the product again by .2618 for the content.

M 5

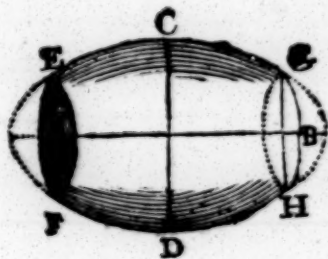
Ex

EXAMPLES.

1. Required the solidity of the middle frustum EGHF of a spheroid, the greatest diameter CD being 30, the diameter of each end EF or GH 18, and the length AB 40.

18	30
18	30
<hr/>	<hr/>
144	900
18	2
<hr/>	<hr/>
324	1800
<hr/>	<hr/>
	324
	<hr/>
	2124
	40
	<hr/>
	84960
	•2618
	<hr/>
	679680
	8496
	50976
	16992
	<hr/>
	222425280
	<hr/>

Answer.



Ex. 2. What is the solidity of the middle frustum of an oblate spheroid, having the diameter of each circular end 40, the middle 50, and the length 18?

Ans. 3110184.

CASE

CASE II.

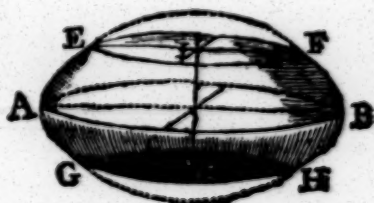
When the Ends are Elliptical, or Perpendicular to the Revolving Axis.

To double the product of the transverse and conjugate diameters of the middle section, add the product of the transverse and conjugate of one end; multiply the sum by the length of the frustum, and the product again by .2618 for the content.

EXAMPLES.

1. In the middle frustum EFHG of an oblate spheroid, the diameters of the middle or greatest elliptic section AB are 50 and 30, and of one end EF or GH 40 and 24; required the content, the height IK being 9?

24	50
<u>40</u>	<u>30</u>
960	1500
<u> </u>	<u>2</u>
	3000
	960
	<u>3960</u>
	9
	<u>35640</u>
	2618
	<u>285120</u>
	3564
	21384
	<u>7128</u>
	<u>93305520 Ans.</u>



Ex. 2. In the middle frustum of an oblate spheroid, the axes of the middle ellipse are 50 and 30, and those of each end are 30 and 18; required the content, the height being 40?

Ans. 37070.88

PROBLEM XVIII.

To find the Solidity of an Elliptic Spindle.

R U L E I.

1. Take the difference between 3 times the square of the middle or greatest diameter, and 4 times the square of the diameter at $\frac{1}{4}$ of the length, or equally distant between the middle and one end; also take the difference between 3 times the greatest diameter, and 4 times the said middle diameter. Then the former difference divided by the latter, will be quadruple the central distance, or distance between the centre of the spindle and centre of the generating ellipse.

2. Then find the axes of the ellipse by problem II, and the area of the segment which generated the spindle by problem v.

3. Divide 3 times that area by the length of the spindle; from the quotient subtract the greatest diameter; and multiply the remainder by 4 times the central distance, before found.

4. Subtract this product from the square of the greatest diameter; and multiply the remainder by the length of the spindle, and again by .5236, for the solidity.

E X A M P L E S.

1. Required the solidity of the elliptic spindle ACBDA, the length AB being 40, the greatest diameter CD 12, and the diameter EF, at $\frac{1}{4}$ of the length, 9.49546?

2. For

2. For the Central Distances, and Axes of the Ellipse.

$$\begin{array}{r} 4 \text{ EF} \quad 37.98184 \\ 3 \text{ CD} \quad 36.00000 \end{array}$$

$$\text{dif.} \quad \underline{1.98184}$$

$$\begin{array}{r} 3 \text{ CD}^2 \quad 432.00000 \\ 4 \text{ EF}^2 \quad 360.6546 \end{array}$$

$$1.98184) 71.3454 (36 = 4 \text{ OG}$$

$$59.4582 \quad 9 = \text{OG}$$

$$6 = \text{CG}$$

$$\underline{118872}$$

$$118916$$

$$15 = \text{OC}$$

$$30 = \text{CH the conj.}$$

$$24 = \text{GH}$$

$$6 = \text{CG}$$

$$\underline{144}$$

its root 12 = mean between CG and GH.

Then as 12 : 20 (or AG) : : 30 (or CH) : 50 = IK
the transverse.

2. For the Generating Elliptic Segment.

$$\text{CH } 30) 6 \text{ CG}$$

$$\underline{\quad} \quad .2 \text{ tab. verf.}$$

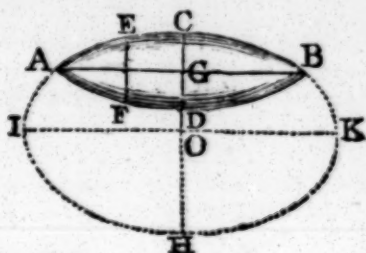
$$.111823 \text{ tab. area corresp.}$$

$$50 \text{ IK}$$

$$\underline{5.591150}$$

$$30 \text{ CH}$$

$$\underline{167.734500} \text{ area generating seg. ACBA.}$$



3. For

3. For the Solidity of the Spindle.

$$\begin{array}{r}
 167.7345 \\
 \underline{3} \\
 \text{AB } 40 \text{) } 503.2035 \\
 \quad 12.5800875 \\
 \text{CD} \quad \quad 12 \\
 \hline
 \quad \quad 0.5800875 \\
 4 \text{ OG} \quad \quad \quad 36 \\
 \hline
 \quad \quad 34805250 \\
 \quad 17402625 \\
 \hline
 \text{prod. from } 20.8831500 \\
 \quad 144. \\
 \hline
 \text{rem. AB} \quad 123.11685 \\
 \quad \quad \quad 40 \\
 \hline
 \quad 4924.67400 \\
 \quad \quad 5236 \\
 \hline
 \quad 29548044 \\
 \quad 14774022 \\
 \quad 9849348 \\
 \quad 24623370 \\
 \hline
 \hline
 \end{array}$$

solidity 2578.5593064 Answer.

Ex. 2. Required the solidity of the elliptic spindle, whose length is 20, the greatest diameter 6, and the diameter at $\frac{1}{4}$ of the length 4.74773? Ans. 322.32.

R U L E II.

To the square of the greatest diameter, add the square of double the diameter at $\frac{1}{4}$ of the length; multiply the sum by the length, and the product again by .1309 for the solidity, very nearly.

Note.

Note. This rule will also serve for any other solid formed by the revolution of any conic section.

EXAMPLE.

What is the solid content of the elliptic spindle whose length is 20, the greatest diameter 6, and the diameter at $\frac{1}{4}$ of the length 4.74773?

$$\begin{array}{r} 4.74773 \\ 2 \end{array}$$

$$\begin{array}{r} 9.49546 \text{ double the diam.} \\ 645949 \text{ ditto inverted} \end{array}$$

$$\begin{array}{r} 8545914 \\ 379818 \\ 85459 \\ 4748 \\ 380 \\ 56 \end{array}$$

$$\begin{array}{r} 90.16375 \text{ sq. of double diam.} \\ 36.00000 \text{ sq. of other diam.} \end{array}$$

$$\begin{array}{r} 126.16375 \text{ sum} \\ 20 \text{ length} \end{array}$$

$$\begin{array}{r} 2523.27500 \\ 9031 \end{array} \quad \text{or } 1309 \text{ inverted}$$

$$\begin{array}{r} 2523 \\ 757 \\ 22 \end{array}$$

Anf. 330.2 the solidity nearly.

PROBLEM XIX.

To find the Solidity of a Frustum or Segment of an Elliptic Spindle.

Proceed as in the last rule, for this, or any other solid formed

formed by the revolution of a conic section about an axis, namely,

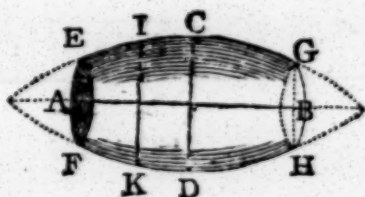
Add together the squares of the greatest and least diameters, and the square of double the diameter in the middle between the two; multiply the sum by the length, and the product again by $\cdot 1309$ for the solidity.

Note. For all such solids, this rule is exact when the body is formed from the conic section, or a part of it, revolved about the axis of the section. And will always be very near the truth when the figure revolves about another line.

EXAMPLES.

1. Required the content of the middle frustum EGHF of any spindle, the length AB being 40, the greatest or middle diameter CD 32, the least or diameter at either end EF or GH 24, and the diameter IK, in the middle between EF and CD, $30\cdot157568$?

32	30 \cdot 157568	
32	2	
64	60 \cdot 315136	double
96	51306	invert
1024	361890	
	1809	
24	60	
24	30	
96	3637 \cdot 89	sq. of 2 IK
48	1024 \cdot 00	sq. of CD
	576 \cdot 00	sq. of EF
576	5237 \cdot 89	sum
	40	length
209515 \cdot 60		
9031		inverted
20951		
6285		
188		
27424	Answer.	



Ex. 2.

Ex. 2. What is the content of the segment of any spindle, the length being 10, the greatest diameter 8, and the middle diameter 6? Ans. 272.272.

Ex. 3. Required the solidity of the frustum of an hyperbolic conoid, the height being 12, the greatest diameter 10, the least diameter 6, and the middle diameter $8\frac{1}{2}$? Ans. 667.59.

Ex. 4. What is the content of the middle frustum of an hyperbolic spindle, the length being 20, the middle or greatest diameter 16, the diameter at each end 12, and the diameter at $\frac{1}{4}$ of the length $14\frac{1}{2}$? Ans. 3248.938.

PROBLEM XX.

To find the Solidity of a Parabolic Conoid.

Square the diameter of the base, multiply that by the altitude, and the product again by .3927, for the content.

EXAMPLES.

1. Required the solidity of the paraboloid whose height BD is 30, and the diameter of its base AC is 40?

$$\begin{array}{r}
 40 \\
 40 \\
 \hline
 1600 \\
 30 \\
 \hline
 48000 \\
 3927 \\
 \hline
 31416 \\
 15708 \\
 \hline
 18849.6000 \text{ Answer.}
 \end{array}$$

Ex. 2.

Ex. 2. What is the content of the parabolic conoid whose altitude is 42, and the diameter of its base 24?

Anf. 9500·1984.

PROBLEM XXI.

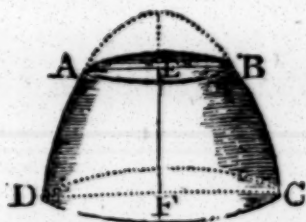
To find the Solidity of the Frustum of a Paraboloid.

Square the diameter of the two ends, add those two squares together, multiply that sum by the height, and the product again by ·3927, for the content.

EXAMPLES.

1. Required the content of the paraboloidal frustum ABCD, the diameter AB being 20, the diameter DC 40, and the height EF $22\frac{1}{2}$?

$$\begin{array}{r}
 1600 \text{ DC}^2 \\
 400 \text{ AB}^2 \\
 \hline
 2000 \text{ sum} \\
 22\frac{1}{2} \text{ EF} \\
 45000 \\
 \cdot 3927 \\
 \hline
 19635000 \\
 15708 \\
 \hline
 176715000 \text{ Anf.}
 \end{array}$$



Ex. 2. What is the content of the frustum of a paraboloid, the greatest diameter being 30, the least 24, and the altitude 9?

Anf. 5216·6266.

PROBLEM XXII.

To find the Solidity of a Parabolic Spindle.

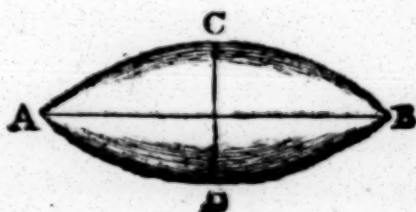
Take the square of the middle or greatest diameter, multiply it by the length, and the product again by ·41888, for the content.

EX-

EXAMPLES.

1. Required the content of the parabolic spindle ACBD, whose length AB is 40, and the greatest diameter CD 16?

$$\begin{array}{r}
 16 \text{ CD} \\
 16 \\
 \hline
 96 \\
 16 \\
 \hline
 256 \text{ CD}^2 \\
 40 \text{ AB} \\
 \hline
 10240 \\
 + 41888 \\
 \hline
 1675520 \\
 83776 \\
 41888 \\
 \hline
 428933120 \text{ Answer.}
 \end{array}$$



Ex. 2. What is the solidity of a parabolic spindle, whose length is 18, and its middle diameter 6 feet?

Ans. 271'4336.

PROBLEM XXIII.

To find the Solidity of the Middle Frustum of a Parabolic Spindle.

Add all together, 8 times the square of the greatest diameter, 3 times the square of the least diameter, and 4 times the product of the two diameters; multiply the sum by the length, and the product again by '05236 for the solidity.

E X-

EXAMPLES.

1. Required the content of the frustum of a parabolic spindle EGHF, the length AB being 20, the greatest diameter CD 16, and the least diameter EF 12?

16	12	16
16	12	12
—	—	—
96	144	192
16	3	4
—	—	—
256	432	768
8	—	—
—	—	—

2048 $8CD^2$
 432 $3EF^2$
 768 $4CD \times EF$

3248 sum
 20 AB

64960
 05236

See fig. in p. 260.

389760
 19488
 12992
 32480

3401.30560 Answer.

Ex. 2. What is the content of the frustum of a parabolic spindle, whose length is 18, greatest diameter 18, and least diameter 10? Ans. 3404.23776.

Note. The solidities of the hyperboloid and hyperbolic spindle, are to be found by rule 2 to prob. XVIII. And those of their frustums by prob. XIX; where some examples of them are given.

O F
G A U G I N G.

THE business of cask-gauging is commonly performed by two instruments, namely, the gauging or sliding-rule, and the gauging or diagonal rod.

I. OF THE GAUGING RULE.

This instrument serves to compute the contents of casks, &c. after the dimensions have been taken. It is a square rule, having various logarithmic lines on its four sides or faces; and three sliding pieces, running in grooves, in three of them.

Upon the first face are three lines, namely, two marked A, B, for multiplying and dividing; and the third, MD, for malt depth, because it serves to gauge malt. The middle one B is on the slider, and is a kind of double line, being marked at both the edges of the slider, for applying it to both the lines A and MD. These three lines are all of the same radius, or distance from 1 to 10; each containing twice the length of the radius. A and B are placed and numbered exactly alike, each beginning at 1, which may be either 1, or 10, or 100. &c. or $\cdot 1$, or $\cdot 01$, or $\cdot 001$, &c. but whatever it is, the middle division, 10, will be ten times as much, and the last division,

100 times as much. But 1 on the line MD is opposite 215, or more exactly 2150.4 on the other lines, which number 2150.4 denotes the cubic inches in a malt bushel; and its divisions numbered retrograde to those of A and B. On these two lines are also several other marks and letters: thus, on the line A are MB, for malt bushel, at the number 2150.4; and A for ale, at 282, the cubic inches in an ale gallon; and on the line B is W, for wine, at 231, the cubic inches in a wine gallon; also *s i*, for square inscribed, at 707, the side of a square inscribed in a circle whose diameter is 1; *s e*, for square equal, at 886, the side of a square which is equal to the same circle; and *c*, for circumference, at 31416, the circumference of the same circle.

On the second face, or that opposite the first, are a slider and four lines, marked I, D, E, at one end, and root, square, root, cube, at the other; the lines C and D containing respectively the squares and cubes of the opposite numbers on the lines D, D; the radius of D being double to that of A, B, C, and triple to that of E; so that whatever the first 1 on D denotes, the first on C is the square of it, and the first on E the cube of it; so if D begin with 1, C and E will begin with 1; but if D begin with 10, C will begin with 100, and E with 1000; and so on. On the line C are marked *o c* at 0796, for the area of the circle, whose circumference is 1; and *o d* at 7854, for the area of the circle whose diameter is 1. Also, on the line D, are WG, for wine gauge, at 1715; and AG for ale gauge, at 1895; and MR, for malt round, at 5232; these three being the gauge points for round and circular measure, and are found by dividing the square roots of 231, 282, and 2150.4, by the square root of 7854; also MS, for malt square, are marked at 4637, the malt gauge point for square measure, being the square root of 2150.4.

On the third face are three lines, one on a slider marked N; and two on the stock, marked SS and SL, for segment standing and segment lying, which serve for ullaging standing and lying casks. And

And on the fourth, or opposite face, are a scale of inches, and three other scales, marked spheroid, or 1st variety, 2d variety, 3d variety; the scale for the 4th, or conic variety, being on the inside of the slider in the third face. The use of these lines is to find the mean diameters of casks.

Besides all those lines, there are two others on the insides of the two first sliders, being continued from the one slider to the other. The one of these is a scale of inches, from $12\frac{1}{2}$ to 36; and the other is a scale of ale gallons, between the corresponding numbers 435 and 361; which form a table to shew, in ale gallons, the contents of all cylinders whose diameters are from $12\frac{1}{2}$ to 36 inches, their common altitude being 1 inch.

The Use of the Gauging Rule.

PROBLEM I.

To Multiply two Numbers, as 12 and 25.

Set 1 on B, to either of the given numbers, as 12, on A; then against 25 on B, stands 300 on A; which is the product.

PROBLEM II.

To Divide one Number by another, as 300 by 25.

Set 1 on B, to 25 on A; then against 300 on A, stands 12 on B, for the quotient.

PROBLEM III.

To find a Fourth Proportional, as to 8, 24, and 96.

Set 8 on B, to 24 on A; then against 96 on B, is 288 on A, the 4th proportional to 8, 24, 96, required.

PROBLEM IV.

To Extract the Square Root, as of 225.

The first 1 on C standing opposite the one on D, on the stock; then against 225 on C, stands its square root 15 on D.

PROBLEM V.

To Extract the Cube Root, as of 3375.

The line D on the slide being set straight with E; then opposite 3375 on E, stands its cube root 15 on D.

PROBLEM VI.

To find a Mean Proportional, as between 4 and 9.

Set 4 on C, to the same 4 on D; then against 9 on C, stands the Mean proportional 6 on D.

PROBLEM VII.

To find Numbers in Duplicate Proportion.

As, to find a Number which shall be to 120, as the Square of 3 to the Square of 2.

Set 2 on D, to 120 on C; then against 3 on D, stands 270 on C, for the answer.

PROBLEM VIII.

To find Numbers in Subduplicate Proportion.

As, to find a Number which shall be to 2 as the Root of 270 to the Root of 120.

Set 2 on D, to 120 on C; then against 270 on C, stands 3 on D, for the answer.

P R O.

PROBLEM IX.

To find Numbers in Triplicate Proportion.

As, to find a Number which shall be to 100, as the Cube of 36 is to the Cube of 40.

Set 40 on D, to 100 on E; then against 36 on D, stands 72.9 on E, for the answer.

PROBLEM X.

To find Numbers in Subtriplicate Proportion.

As, to find a Number which shall be to 40, as the Cube Root of 72.9 is to the Cube Root of 100.

Set 40 on D, to 100 on E; then against 72.9 on E, stands 36 on D, for the answer.

PROBLEM XI.

To Compute Malt Bushels by the Line MD.

As, to find the Malt Bushels in the Couch, Floor, or Cistern, whose Length is 230, Breadth 58.2, and Depth 5.4 Inches.

Set 230 on B, to 5.4 on MD; then against 58.2 on A, stands 33.6 bushels on B, for the answer.

Note. The uses of the other marks on the rule, will appear in the examples farther on.

OF THE GAUGING OR DIAGONAL ROD.

The diagonal rod is a square rule, having four faces; being commonly 4 feet long, and folding together by joints.

N

joints. This instrument is used both for gauging or measuring casks, and computing their contents, and that from one dimension only, namely the diagonal of the cask, or the length from the middle of the bung hole to the meeting of the head of the cask with the stave opposite to the bung; being the longest line that can be drawn within the cask from the middle of the bung. And, accordingly, on one face of the rule is a scale of inches for measuring this diagonal; to which are placed the areas, in ale gallons, of circles to the corresponding diameters, in like manner as the lines on the under sides of the three slides in the sliding rule.

On the opposite face, are two scales of ale and wine gallons, expressing the contents of casks having the corresponding diagonals. And these are the lines which chiefly form the difference between this instrument and the sliding rule; for all their other lines are the same, and are to be used in the same manner.

EXAMPLE.

The rod being applied within the cask at the bung-hole, the diagonal was found to be 34'4 inches; required the content in gallons.

Now to 34'4 inches correspond, on the rod, $90\frac{1}{4}$ ale gallons, or 111 wine gallons, the content required.

Note. The contents exhibited by the rod, answer to the most common form of casks, and fall in between the 2d and 3d varieties following.

OF CASKS AS DIVIDED INTO VARIETIES.

It is usual to divide casks into four cases or varieties, which are judged of from the greater or less apparent curvature of their sides; namely,

1. The middle frustum of a spheroid,
2. The middle frustum of a parabolic spindle,
3. The two equal frustums of a paraboloid,
4. The two equal frustums of a cone.

And

And if the content of any of these be computed in inches, by their proper rules, and this be divided by 282, or 231, or 2150.4, the quotient will be the content in ale gallons, or wine gallons, or malt bushels, respectively. Because

282	cubic inches make	1	ale gallon
231	- - - - -	1	wine gallon
2150.4	- - - - -	1	malt bushel.

And the particular rule will be for each as in the following problems :

PROBLEM XII.

To find the Content of a Cask of the First Form.

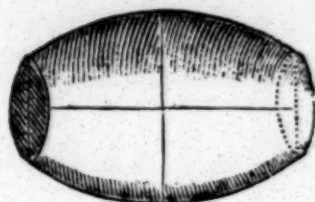
To the square of the head diameter, add double the square of the bung diameter; and multiply the sum by the length of the cask. Then let the product

be multiplied by $\cdot 0009\frac{2}{3}$, or divided by 1077, for ale gallons;
and multiplied by $\cdot 0011\frac{1}{3}$, or divided by 882, for wine gallons.

EXAMPLES.

1. Required the content of a spheroidal cask, whose length is 40, and bung and head diameters 32 and 24 inches.

24	32
24	32
—	—
96	64
48	96
—	—
576	1024
—	—
	2



2048	104960	104960
576	•0009 $\frac{1}{4}$	•0011 $\frac{1}{3}$
—	—	—
2624	944640	1154560
40	26240	34987
—	—	—
104960	ale 97•0880	gallons 118•9547
—	—	—
		wine

By the Gauging Rule.

Having set 40 on C, to the ale gauge 32•82 on D,
against

24 on D, stands 21•3 on C

32 on D, stands 38•0 on C

the same 38•0

—
sum 97•3 ale gallons.

And having set 40 on C, to the wine gauge 29•7 on D,
against

24 on D, stands 26•1 on C

32 on D, stands 46•5 on C

the same 46•5

—
sum 119•1 wine gallons.

Ex. 2. Required the content of the spheroidal cask;
whose length is 20, and diameters 12 and 16 inches.

Answer { 12•136 ale gallons,
 { 14•869 wine gallons.

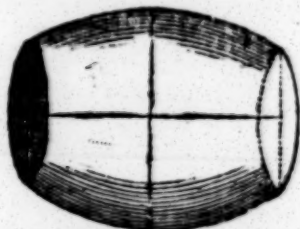
PROBLEM XIII.

To find the Content of a Cask of the Second Form.

To the square of the head diameter, add double the square of the bung diameter, and from the sum take $\frac{2}{5}$ or $\frac{4}{10}$ of the square of the difference of the diameters; then multiply the remainder by the length, and the product again by $\cdot 0009\frac{1}{4}$ for ale gallons, or by $\cdot 0011\frac{1}{3}$ for wine gallons.

EXAMPLES.

1. The length being 40, and diameters 24 and 32, required the content?



32			
24			
—			
8	2624·0	103936	103936
8	25·6	·0009 $\frac{1}{4}$	·0011 $\frac{1}{3}$
—	—	—	—
64	2598·4	935424	1143296
4	40	25984	34645
—	—	—	—
25·6	103936	ale 96·1408 gall.	117·7941 wine
—	—	—	—

By the Gauging Rule.

Having set 40 on C, to 32·82 on D, against 8 on D, stands 2·4 on C; the $\frac{4}{10}$ of which is 0·96. This taken from the 97·3 in the last form, leaves 96·3 ale gallons.

N 3

And

And having set 40 on C, to 29.7 on D, against 8 on D, stands 2.9 on C; the $\frac{4}{10}$ of which is 1.16. This taken from the 119.1 in the last form, leaves 117.9 wine gallons.

Ex. 2. Required the content when the length is 20, and the diameters 12 and 16.

Answer $\left\{ \begin{array}{l} 12.018 \text{ ale gallons,} \\ 14.724 \text{ wine gallons.} \end{array} \right.$

PROBLEM XIV.

To find the Content of a Cask of the Third Form.

To the square of the bung diameter, add the square of the head diameter; multiply the sum by the length, and the product again by .0014 for ale gallons, or by .0017 for wine gallons.

EXAMPLES.

1. Required the content of a cask of the third form, when the length is 40, and the diameters 24 and 32.



1024	64000	64000
576	.0014	.0017
<hr/>	<hr/>	<hr/>
1600	256	449
40	64	64
<hr/>	<hr/>	<hr/>
64000	89.6	108.8
<hr/>	<hr/>	<hr/>
	ale	gallons
		wine

By

By the Gauging Rule.

Set 40 on C, to 26.8 on D; then against
 24 on D, stands 32.0 on C
 32 on D, stands 57.3 on C

sum 89.3 ale gallons.

And having set 40 on C, to 24.25 on D; then against
 24 on D, stands 39.1 on C
 32 on D, stands 69.8 on C

sum 108.9 wine gallons.

Ex. 2. Required the content when the length is 20, and the diameters 12 and 16.

Answer { 11.2 ale gallons,
 13.6 wine gallons.

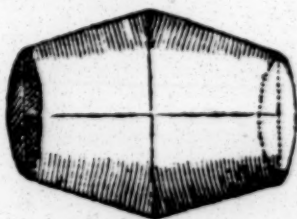
PROBLEM XV.

To find the Content of a Cask of the Fourth Form.

Add the square of the difference of the diameters, to 3 times the square of their sum; then multiply the sum by the length, and the product again by $.00023\frac{1}{3}$ for ale gallons, or by $.00028\frac{1}{3}$ for wine gallons.

EXAMPLES.

1. Required the content, when the length is 40, and the diameters 24 and 32 inches.



56	8		
56	8		
<hr/>	<hr/>	378880	378880
336	64	00023 $\frac{1}{3}$	00028 $\frac{1}{3}$
280	9408	<hr/>	<hr/>
<hr/>	<hr/>	1136640	3031040
3136	9472	757760	757760
3	40	75776	126293
<hr/>	<hr/>	<hr/>	<hr/>
9408	378880 ale	87.90016 gall.	107.34933 wine
<hr/>	<hr/>	<hr/>	<hr/>

By the Sliding Rule.

Set 40 on C, to 65.64 on D; then against

8 on D, stands 0.6 on C

56 on D, stands 29.1 on C

29.1

29.1

sum 87.9 ale gallons.

And set 40 on C, to 59.41 on D; then against

8 on D, stands 0.7

56 on D, stands 35.6

35.6

35.6

sum 107.5 wine gal.

Ex. 2. What is the content of a conical cask, the length being 20, and the bung and head diameters 16 and 12 inches?

Answer { 10.985 ale gallons,
13.416 wine gallons.

PROBLEM XVI.

To find the Content of a Cask by Four Dimensions.

Add together, the squares of the bung and head diameters, and the square of double the diameter taken in the middle between the bung and head; then multiply the sum by the length of the cask, and the product again by $\cdot 0004\frac{2}{3}$ for ale gallons, or by $\cdot 0005\frac{2}{3}$ for wine gallons.

EXAMPLES.

1. Required the content of any cask whose length is 40, the bung diameter being 32, the head diameter 24, and the middle diameter between the bung and head $28\frac{3}{4}$ inches.

57.5	24	32
57.5	24	32
<hr/>	<hr/>	<hr/>
2875	96	64
4025	48	96
2875	<hr/>	<hr/>
<hr/>	576	1024
3306.25	<hr/>	<hr/>
1024		
576		
<hr/>		
4906.25		
40		
<hr/>		
196250	196250	
$\cdot 0004\frac{2}{3}$	$\cdot 0005\frac{2}{3}$	
<hr/>	<hr/>	
785000	981250	
130833	130833	
<hr/>	<hr/>	
ale 91.5833	gallons 111.2083	wine
<hr/>	<hr/>	

N 5

By

By the Sliding Rule.

Set 40 on C, to 46.4 on D; then against
 24 on D, stands 10.5
 32 on D, stands 13.0
 57½ on D, stands 62.0

sum 91.5 ale gallons.

Set 40 on C, to 42.0 on D; then against
 24 on D, stands 13.0
 32 on D, stands 23.2
 57½ on D, stands 75.0

sum 111.2 wine gallons.

Ex 2. What is the content of a cask, whose length is 20, the bung diameter being 16, the head diameter 12, and the diameter in the middle between them 14½?

Answer { 11.4479 ale gallons,
 { 13.9010 wine gallons.

PROBLEM XVII.

To find the Content of any Cask from Three Dimensions only.

Add into one sum, 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the two diameters: then multiply the sum by the length, and the product again by $\frac{00034}{9}$ for wine gallons, or by $\frac{00034}{11}$ or $00003\frac{1}{11}$ for ale gallons.

EXAMPLES.

1. Required the content of a cask, whose length is 40 and the bung and head diameters 32 and 24?

6

32

32	24	32
<u>32</u>	<u>24</u>	<u>24</u>
61	96	128
<u>56</u>	<u>48</u>	<u>64</u>
1024	576	768
<u>39</u>	<u>25</u>	<u>26</u>
9216	2880	4608
<u>3072</u>	<u>1152</u>	<u>1536</u>
39936	14400	19968
	<u>39936</u>	
	19968	
	<u>74304</u>	
	40	
	<u>2972160</u>	
	'00034	2972160
	<u>11888640</u>	'00003 $\frac{1}{4}$
	8916480	<u>8916480</u>
		270196
9) 1010'53440		
	112'2816 wine gal.	91'86676 ale gallons.

Ex. 2. What is the content of a cask, whose length is 20, and the bung and head diameters 16 and 12?

Answer { 11'4833 ale gallons,
14'0352 wine gallons.

Note. This is the most exact rule of any, for three dimensions only; and agrees nearly with the diagonal rod.

OF THE ULLAGE OF CASKS.

The ullage of a cask, is what it contains when only partly filled. And it is considered in two positions, namely, as standing on its end with the axes perpendicular to the horizon, or as lying on its side with the axes parallel to the horizon.

PROBLEM XVIII.

To find the Ullage by the Sliding Rule.

By one of the preceding problems find the whole content of the cask. Then set the length on N, to 100 on SS, for a segment standing, or set the bung diameter on N, to 100 on SL, for a segment lying; then against the wet inches on N, is a number on SS or SL, to be reserved.

Next, set 100 on B, to the reserved number on A; then against the whole content on B, will be found the ullage on A.

EXAMPLES.

1. Required the ullage answering to 10 wet inches of a standing cask, the whole content of which is 92 gallons, and length 40 inches;

Having set 40 on N, to 100 on SS; then
against 10 on N, is 23 on SS, the reserved numb.

Then set 100 on B, to 23 on A; and
against 92 on B, is 21.2 on A, the ullage required?

Ex. 2. What is the ullage of a standing cask whose whole length is 20 inches, and content $11\frac{1}{2}$ gallons; the wet inches being 5?

Ans. 2.65 gallons.

Ex. 3.

Ex. 3. The content of a cask being 92 gallons, and the bung diameter 32, required the ullage of the segment lying when the wet inches are 8?

Ans. 16.4 gallons.

P R O B L E M X I X.

To Ullage a Standing Cask by the Pen.

Add all together, the square of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the middle between the other two; then multiply the sum by the length between the surface and nearest end, and the product again by $\cdot 0004\frac{2}{3}$ for ale gallons, or by $\cdot 0005\frac{2}{3}$ for wine gallons, in the left part of the cask, whether empty or filled.

E X A M P L E.

The three diameters being 24, 27, and 29 inches, required the ullage for 10 wet inches.

24	29	54	
24	29	54	2916
—	—	—	841
96	261	216	576
48	58	270	—
—	—	—	4333
576	841	2916	10
—	—	—	—
	43330		43330
	$\cdot 0004\frac{2}{3}$		$\cdot 0005\frac{2}{3}$
	—		—
	173320		216650
	28885		28885
	—		—
ale 20.2205	gallons	24.5535	wine
—	—	—	—

P R O-

PROBLEM XX.

To Ullage a Lying Cask by the Pen.

Divide the wet inches by the bung diameter; find the quotient in the column of versed sines, in the table of circular segments at the end of the book, taking out its corresponding segment. Then multiply this segment by the whole content of the cask, and the product again by $1\frac{1}{4}$ for the ullage required, nearly.

EXAMPLES.

Supposing the bung diameter 32, and content 92 ale gallons; to find the ullage for 8 wet inches.

$$\begin{array}{r}
 32 \) \ 8 \cdot 25, \text{ whose tab. seg. is } \cdot 153546 \\
 \underline{\hspace{1.5cm}} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} 92 \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} \underline{\hspace{1.5cm}} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} 307092 \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} \underline{\hspace{1.5cm}} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} 1381914 \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} \underline{\hspace{1.5cm}} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} 14 \cdot 126232 \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} \frac{1}{4} \text{ is } \underline{\hspace{1.5cm}} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} 3 \cdot 531558 \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} \underline{\hspace{1.5cm}} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} 17 \cdot 657790 \text{ Answer.} \\
 \phantom{32 \) \ 8 \cdot 25, \text{ whose tab. seg. is }} \underline{\hspace{1.5cm}}
 \end{array}$$

SPECIFIC GRAVITY.

THE specific gravities of bodies, are their relative weights, contained under the same given magnitude, as a cubic foot, or a cubic inch, &c.

The specific gravities of several sorts of matter are expressed by the numbers annexed to their names in the following table :

A Table of the Specific Gravities of Bodies.

Fine gold	19640	Brick	2000
Standard gold	18888	Light earth	1984
Quick-silver	13600	Solid gunpowder	1745
Lead	11325	Sand	1520
Fine silver	11091	Pitch	1150
Standard silver	10535	Box-wood	1030
Copper	9000	Sea-water	1030
Gun metal	8784	Common water	1000
Cast brass	8000	Oak	925
Steel	7850	Gun-powder, shaken	922
Iron	7645	Ash	800
Cast iron	7425	Maple	755
Tin	7320	Elm	600
Marble	2700	Fir	550
Common stone	2520	Cork	240
Loom	2160	Air	$1\frac{1}{4}$

Note. The several sorts of wood are supposed to be dry. Also, as a cubic foot of water weighs just 1000 ounces avoirdupois, the numbers in this table express, not only the specific gravities of the several bodies, but also the weight of a cubic foot of each, in avoirdupois ounces ; and hence, by proportion, the weight of any other quantity, or the quantity of any other weight, may be known, as in the following problems.

P R O.

PROBLEM I.

To find the Magnitude of any Body from its Weight.

As the tabular specific gravity of the body,
Is to its weight in avoirdupois ounces,
So is one cubic foot, or 1728 cubic inches,
To its content in feet, or inches, respectively.

EXAMPLES.

1. Required the content of an irregular block of common stone which weighs 1 cwt. or 112lb.

$$\begin{array}{r}
 112\text{lb} \\
 16 \\
 \hline
 672 \\
 112 \\
 \hline
 2520 : 1792 :: 1728 : 1228\frac{2016}{2520} \\
 1728 \\
 \hline
 14336 \\
 3584 \\
 12544 \\
 1792 \\
 \hline
 2520) 3096576 \text{ (} 1228\frac{2016}{2520} \text{ Answer.} \\
 252 \\
 \hline
 576 \\
 504 \\
 \hline
 725 \\
 504 \\
 \hline
 2217 \\
 2016 \\
 \hline
 2016 \\
 \hline
 \end{array}$$

Ex. 2.

- Ex. 2. How many cubic inches of gun-powder are there in 1 lb. weight? Ans. 30 cubic inches nearly.
 Ex. 3. How many cubic feet are there in a ton weight of dry oak? Ans. $38\frac{1}{8}$ cubic feet.

PROBLEM II.

To find the Weight of a Body from its Magnitude.

As one cubic foot, or 1728 cubic inches,
 Is to the content of the body,
 So is its tabular specific gravity,
 To the weight of the body.

EXAMPLES.

1. Required the weight of a block of marble, whose length is 63 feet, and breadth and thickness each 12 feet; being the dimensions of one of the stones in the walls of Balbeck,

63
 12

—
 756
 12

— oz tons
 1 : 9072 :: 2700 : $683\frac{4}{10}$
 2700

—
 6350400
 18144

16 { 4 | 24494400 oz
 4 | 6123600
 112 | 1530900 lb
 20 | 1368 cwt

Ans. $683\frac{4}{10}$ ton, almost equal to the burthen of an East India ship.

Ex. 2.

Ex. 2. What is the weight of 1 pint, ale measure, of gun-powder? Ans. 19 oz. nearly.

Ex. 3. What is the weight of a block of dry oak, which measures 10 feet long, 3 feet broad, and $2\frac{1}{2}$ feet deep?

Ans. $4335\frac{1}{8}$ lb.

PROBLEM III.

To find the Specific Gravity of a Body.

CASE 1. When the body is heavier than water, weigh it both in water and out of water, and take the difference, which will be the weight lost in water. Then say,

As the weight lost in water,
Is to the whole weight,
So is the specific gravity of water,
To the specific gravity of the body

EXAMPLE.

A piece of stone weighed 10lb, but in water only $6\frac{1}{4}$ lb, required its specific gravity?

$$\begin{array}{r} 10 \\ 6\frac{1}{4} \\ \hline 3\frac{1}{4} : 10 :: 1000 : \\ \text{or } 13 : 40 :: 1000 : 3077 \text{ answer.} \end{array}$$

$$\begin{array}{r} 40 \\ \hline 13 \) \ 40000 \ (\ 3077 \\ \underline{39} \\ 100 \\ \underline{91} \\ 90 \\ \underline{} \end{array}$$

CASE 2. When the body is lighter than water, so that it will not quite sink; affix to it a piece of another body

body heavier than water, so that the mass compounded of the two may sink together. Weigh the heavier body, and the compound mass, separately, both in water and out of it; then find how much each loses in water, by subtracting its weight in water from its weight in air; and subtract the less of these remainders from the greater. Then say,

As this last remainder,
Is to the weight of the light body in air,
So is the specific gravity of water,
To the specific gravity of the body.

EXAMPLE.

Suppose a piece of elm weighs 15lb in air, and that a piece of copper, which weighs 18lb in air, and 16lb in water, is affixed to it, and that the compound weighs 8lb in water; required the specific gravity of the elm?

Copper		Compound
18	in air	33
16	in water	6
<hr/>		
2	loss	27
<hr/>		2
<hr/>		

Then As 25 : 15 :: 100 : 600 ans.

PROBLEM IV.

To find the Quantities of Two Ingredients in a Given Compound.

Take the three differences of every pair of the three specific gravities, namely, the specific gravities of the com-

compound and each ingredient; and multiply the difference of every two specific gravities by the third. Then, as the greatest product is to the whole weight of the compound, so is each of the other products, to the two weights of the ingredients.

EXAMPLE.

A composition of 112lb being made of tin and copper, whose specific gravity is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and of copper 9000?

9000	9000	8784
7320	8784	7320
<hr/>	<hr/>	<hr/>
1680	216	1464 diff.
8784	7320	9000
<hr/>	<hr/>	<hr/>
	4320	13176000
	648	
702720	1512	
52704	<hr/>	
8784	1581120	
<hr/>	<hr/>	
14757120 : 112 : :	13176000 : 100 copper	112
	112	<hr/>
	<hr/>	12 tin
	26352000	
	13176	
	13176	
	<hr/>	

14757120) 1475712000 (100
 Answer, there is 100lb of copper, } in the composition.
 and consequently 12lb of tin }

OF THE
WEIGHT AND DIMENSIONS
OF
BALLS AND SHELLS.

THE weight and dimensions of balls and shells might be found from the problems last given, concerning specific gravity. But they may be found still easier by means of the experimented weight of a ball of a given size, from the known proportion of similar figures, namely, as the cubes of their diameters.

P R O B L E M I.

To find the Weight of an Iron Ball, from its Diameter.

An iron ball of 4 inches diameter weighs 9lb, and the weights being as the cubes of the diameters, it will be, as 64 (which is the cube of 4) is to 9, so is the cube of the diameter of any other ball, to its weight. Or take $\frac{9}{64}$ of the cube of the diameter, for the weight. Or take $\frac{1}{8}$ of the cube of the diameter, and $\frac{1}{8}$ of that again, and add the two together, for the weight.

E X-

E X A M P L E S.

1. The diameter of an iron shot being 6.7, required its weight?

$$\begin{array}{r}
 6.7 \\
 6.7 \\
 \hline
 469 \\
 402 \\
 \hline
 44'89 \\
 6.7 \\
 \hline
 31423 \\
 26934 \\
 \hline
 8) 300.763 \\
 \hline
 8) 37.595 \\
 4.699 \\
 \hline
 \hline
 \text{Ans. } 42.294 \text{ lbs.}
 \end{array}$$

Ex. 2. What is the weight of an iron ball whose diameter is 5.54 inches?

Ans. 24lb.

P R O B L E M II.

To find the Weight of a Lead Ball,

A leaden ball of $4\frac{1}{4}$ inches diameter weighs 17lb; therefore as the cube of $4\frac{1}{4}$ to 17, or nearly as 9 is to 2, so is the cube of the diameter of a leaden ball, to its weight.

Or take $\frac{2}{9}$ of the cube of the diameter, for the weight, nearly.

E X-

EXAMPLES.

1. Required the weight of a leaden ball of 6.6 inches diameter?

$$\begin{array}{r}
 6.6 \\
 6.6 \\
 \hline
 396 \\
 396 \\
 \hline
 43.56 \\
 6.6 \\
 \hline
 26136 \\
 26136 \\
 \hline
 287.496 \\
 2 \\
 \hline
 9 \overline{) 574.992} \\
 \text{Ans. } 63.888 \text{ lb}
 \end{array}$$

Ex. 2. What is the weight of a leaden ball of 5.24 inches diameter? Ans. 32lb nearly.

PROBLEM III.

To find the Diameter of an Iron Ball.

Multiply the weight by $7\frac{1}{2}$, then take the cube root of the product for the diameter.

EXAMPLES.

1. Required the diameter of a 42lb iron ball?

$$\begin{array}{r}
 42 \\
 7\frac{1}{2} \\
 \hline
 294 \\
 4\cdot666 \\
 \hline
 298\cdot666 \text{ or } \frac{2}{3}
 \end{array}$$

The cube root of this is almost 7. Suppose 7, whose cube is 343. Then, by the 2d rule for the cube root at page 41, proceed thus:

$$\begin{array}{r}
 343 \\
 2 \\
 \hline
 686 \\
 298\frac{2}{3} \\
 \hline
 \text{As } 984\frac{2}{3} : 940\frac{1}{3} :: 7 \\
 3 \qquad \qquad 3 \\
 \hline
 \text{Or as } 2954 : 2821 :: 7 : 6\cdot685 \text{ Anf.}
 \end{array}$$

$$\begin{array}{r}
 7 \\
 \hline
 2954 \) \ 19747 \ (\ 6\cdot685 \text{ inches} \\
 \dots \quad 17724 \\
 \hline
 2023 \\
 1772 \\
 \hline
 251 \\
 236 \\
 \hline
 15 \\
 15 \\
 \hline
 \end{array}$$

Ex. 2. What is the diameter of a 24lb iron ball?

Anf. 5\cdot54 inches.

P R O-

PROBLEM IV.

To find the Diameter of a Leaden Ball.

Multiply the weight by 9, and divide the product by 2; then take the cube root of the quotient for the diameter.

EXAMPLES.

1. Required the diameter of a 64lb leaden ball?

$$\begin{array}{r} 64 \\ 9 \\ \hline 2 \ 576 \\ 288 \\ \hline \end{array}$$

the cube root of which is almost 7, whose cube is 343.

$$\begin{array}{r} \text{Then } 343 \\ 2 \\ \hline 686 \\ 288 \\ \hline \end{array} \quad \begin{array}{r} 288 \\ 2 \\ \hline 576 \\ 343 \\ \hline \end{array}$$

$$\text{As } 974 : 919 :: 7 : 6.605 \text{ Anf.}$$

$$\begin{array}{r} 974 \ 6433 \ (6.605 \text{ inches} \\ 5844 \\ \hline 589 \\ 584 \\ \hline 5 \\ \hline \end{array}$$

Ex. 2. What is the diameter of an 8lb leaden ball?

Anf. 3.303 inches.

O

PRO

PROBLEM V.

To find the Weight of an Iron Shell.

Take $\frac{2}{3}$ of the difference of the cubes of the external and internal diameter, for the weight of the shell.

That is, from the cube of the external diameter take the cube of the internal diameter, multiply the remainder by 9, and divide the product by 64.

EXAMPLES.

1. The outside diameter of an iron shell being 12.8, and the inside diameter 9.1 inches; required its weight?

9.1	12.8
9.1	12.8
<hr/> 91	<hr/> 1024
819	1536
<hr/> 82.81	<hr/> 163.84
9.1	12.8
<hr/> 8281	<hr/> 131072
74529	196608
<hr/> 753.571	<hr/> 2097.152
	753.571
	<hr/> 1343.581
	9
	<hr/>
64 { 8)	12092.229
64 { 8)	1511.528
Anf.	188.941 lb.

Ex. 2. What is the weight of an iron shell, whose external and internal diameters are 9.8 and 7 inches?

Anf. $84\frac{1}{4}$ lb.

P R O -

PROBLEM VI.

To find how much Powder will fill a Shell.

Take the cube of the internal diameter, in inches, and divide it by 57·3, for the lbs of powder.

EXAMPLES.

1. How much powder will fill the shell whose internal diameter is 9·1 inches?

$$\begin{array}{r}
 9\cdot1 \\
 9\cdot1 \\
 \hline
 91 \\
 819 \\
 \hline
 82\cdot81 \\
 9\cdot1 \\
 \hline
 8281 \\
 74529 \\
 \hline
 \text{lb} \\
 57\cdot3 \overline{) 753\cdot571} \quad (13 \frac{2}{3} \text{ nearly} \\
 \underline{573} \\
 1805 \\
 \underline{1719} \\
 86 \\
 \hline
 \end{array}$$

Ex. 2. How much powder will fill the shell whose internal diameter is 7 inches?

Ans. 6lb.

PROBLEM VII.

To find how much Powder will fill a Rectangular Box.

Find the content of the box in inches, by multiplying the length, breadth, and depth all together. Then divide by 30, for the pounds of powder.

EXAMPLES.

1. Required the quantity of powder that will fill a box; the length being 15 inches, the breadth 12, and the depth 10 inches?

$$\begin{array}{r}
 15 \\
 12 \\
 \hline
 180 \\
 10 \\
 \hline
 30 \overline{) 1800} \\
 \hline
 \text{Ans. } 60 \text{ lb.} \\
 \hline
 \end{array}$$

Ex. 2. How much powder will fill a cubical box, whose side is 12 inches?

Ans. $57\frac{2}{3}$ lb.

PROBLEM VIII.

To find how much Powder will fill a Cylinder.

Multiply the square of the diameter by the length; then divide by 38.2, for the pounds of powder.

EXAMPLES.

1. How much powder will the cylinder hold, whose diameter is 10 inches, and length 20 inches?

10

$$\begin{array}{r}
 10 \\
 10 \\
 \hline
 100 \\
 20 \\
 \hline
 \text{lb.} \\
 38.2 \times 2000 \left(52\frac{1}{3} \text{ nearly} \right) \\
 1910 \\
 \hline
 900 \\
 764 \\
 \hline
 136 \\
 \hline
 \end{array}$$

Ex. 2. How much powder can be contained in the cylinder, whose diameter is 4 inches, and length 12 inches?
 Ans. $51\frac{1}{2}$ lb.

PROBLEM IX.

To find the Size of a Shell to contain a Given Weight of Powder.

Multiply the pounds of powder by 57.3; then take the cube root of the product, for the diameter in inches.

EXAMPLES.

1. What is the diameter of a shell that will hold $13\frac{1}{2}$ lb. of powder?

$$\begin{array}{r}
 57.3 \\
 13\frac{1}{2} \\
 \hline
 1719 \\
 573 \\
 955 \\
 \hline
 754.45 \\
 03
 \end{array}$$

The

The cube root of this is nearly 9, whose cube is 729.

Then	
729	754'45
2	2
-----	-----
1458	1508'90
754'45	729'
-----	-----

As 2212'45 : 2237'90 :: 9 : 9'1 Answer.

$$\begin{array}{r}
 9 \\
 \hline
 221,245 \) \ 2014,10 \ (\ 9'1 \text{ inches} \\
 \underline{1991} \\
 23 \\
 \underline{22} \\
 1 \\
 \hline
 \end{array}$$

Ex. 2. What is the diameter of a shell, to contain 6lb of powder? Ans. 7 inches.

PROBLEM X.

To find the Size of a Cubical Box, to contain a Given Weight of Powder.

Multiply the weight in pounds by 30, then the cube root of the product will be the side of the box in inches.

EXAMPLES.

1. Required the size of a cubical box, to hold 50lb of gun-powder?

50
30

The cube root of 1500 is 11 nearly, whose cube is 1331.

Then 1331 1500

2 2

2662 3000

1500 1331

As 4162 : 4331 :: 11 : 11.44 Anf.

4162) 47641 (11.44 inches.

.. 45782

1859

1665

194

Ex. 2. Required the size of a cubical box, to hold 400lb of gun-powder? Anf. 22.89 inches.

PROBLEM XI.

To find what Length of a Cylinder will be filled by a Given Weight of Gun-powder.

Multiply the weight in pounds by 38.2; then divide the product by the square of the diameter in inches, for the length.

EXAMPLES.

1. What length of a 36 pounder gun, of $6\frac{1}{2}$ inches diameter, will be filled with 12lb of powder?

0 4

62

$$\begin{array}{r}
 6\frac{2}{3} = \frac{20}{3} \quad 38\frac{2}{12} \\
 \text{its sq.} = 4\frac{00}{9} \quad 12 \\
 \hline
 458\frac{4}{9} \\
 \hline
 400 \overline{) 4125\cdot6} \\
 \hline
 \text{Ans. } 10\cdot314 \text{ inches} \\
 \hline
 \end{array}$$

Ex. 2. What length of a cylinder of 8 inches diameter may be filled with 20lb of powder? Ans. $11\frac{1}{8}$ in.



OF THE

PILING

OF

BALLS AND SHELLS.

IRON balls and shells are commonly piled, by horizontal courses, either in a pyramidal or wedge-like form; the base being either an equilateral triangle, a square, or a rectangle. In the triangle and square, the pile will finish in a single ball; but in the rectangle it will finish in a single row of balls, like an edge.

In triangular and square piles, the number of horizontal rows, or courses, is always equal to the number of balls in one side of the bottom row. And in rectangular piles, the

the number of rows is equal to the number of balls in the breadth of the bottom row. Also the number in the top row or edge, is one more than the difference between the length and breadth of the bottom row.

PROBLEM I.

To find the Number of Balls in a Triangular Pile.

Multiply continually together, the number in one side of the bottom row, that number increased by 1, and the same number increased by 2; then $\frac{1}{6}$ of the last product will be the answer.

EXAMPLES.

1. Required the number of balls in a triangular pile, each side of the base containing 30 balls?

$$\begin{array}{r}
 32 \\
 31 \\
 \hline
 32 \\
 96 \\
 \hline
 992 \\
 30 \\
 \hline
 6 \overline{) 29760} \\
 \text{Ans. } 4960
 \end{array}$$

Ex. 2. How many balls are in the triangular pile, each side of the base containing 20?

Ans. 1540.

PROBLEM II.

To find the Number of Balls in a Square Pile.

Multiply continually together, the number in one side of the bottom course, that number increased by 1, and
 $\frac{1}{2}$ double

double the same number increased by 1; then take $\frac{1}{6}$ of the last product for the answer.

EXAMPLES.

1. How many balls are in a square pile of 30 rows?

$$\begin{array}{r}
 61 \\
 31 \\
 \hline
 61 \\
 183 \\
 \hline
 1891 \\
 30 \\
 \hline
 6 \overline{) 56730} \\
 \text{Ans. } 9455
 \end{array}$$

- Ex. 2. How many balls are there in a square pile of 20 rows?

Ans. 2870.

PROBLEM III.

To find the Number of Balls in a Rectangular Pile.

From 3 times the number in the length of the base row, subtract one less than the breadth of the same, multiply the remainder by the said breadth, and the product by one more than the same; and divide by 6 for the answer.

EXAMPLES.

1. Required the number of ball's in a rectangular pile, the length and breadth of the base row being 46 and 15?

46

$$\begin{array}{r}
 46 \\
 3 \\
 \hline
 138 \\
 14 \\
 \hline
 124 \\
 15 \\
 \hline
 620 \\
 124 \\
 \hline
 1860 \\
 16 \\
 \hline
 11160 \\
 1860 \\
 \hline
 6 \overline{) 29760} \\
 \text{Ans. } 4960
 \end{array}$$

Ex. 2. How many shot are in a rectangular complete pile, the length of the bottom course being 59, and its breadth 20? Ans. 11060.

PROBLEM IV.

To find the Number of Balls in an Incomplete Pile.

From the number in the whole pile, considered as complete, subtract the number in the upper pile which is wanting at the top, both computed by the rule for their proper form; and the remainder will be the number in the frustum, or incomplete pile.

EXAMPLES.

1. To find the number of shot in the incomplete triangular

angular pile, one side of the bottom course being 40, and the top course 20.

19	40
20	41
<hr/>	<hr/>
380	1640
21	42
<hr/>	<hr/>
380	3280
760	6560
<hr/>	<hr/>
7980	68880
<hr/>	<hr/>
	7980
	<hr/>
	6) 60900
	10150 Answer.
	<hr/>

Ex. 2. How many shot are in the incomplete triangular pile, the side of the base being 24, and of the top 7? Ans. 2516.

Ex. 3. How many balls are in the incomplete square pile, the side of the base being 24, and of the top 17? Ans. 4760.

Ex. 4. How many shot are in the incomplete rectangular pile, of 12 courses, the length and breadth of the base being 40 and 20? Ans. 6185.

OF
DISTANCES
BY THE
VELOCITY OF SOUND.

BY various experiments it has been found that sound flies through the air, uniformly at the rate of about 1142 feet in one second of time, or a mile in $4\frac{3}{4}$ seconds. And therefore by proportion, any distance may be found corresponding to any given time; namely, multiply the given time in seconds, by 1142, for the corresponding distance in feet; or take $\frac{3}{4}$ of the given time, for the distance in miles.

Note. The time for the passage of sound, in the interval between seeing the flash of a gun, or lightning, and hearing the report, may be observed by a watch or a small pendulum. Or, it may be observed by the beats of the pulse in the wrist, counting on an average, about 70 to a minute in persons in moderate health, or $5\frac{1}{2}$ pulsations to a mile, and more or less according to circumstances.

EXAMPLES.

1. After observing a flash of lightning, it was 12 seconds before I heard the thunder; required the distance of the cloud from whence it came?

$$\begin{array}{r}
 12 \\
 3 \\
 \hline
 14 \) \ 36 \ (\ 2\frac{4}{7} \text{ miles, the answer.} \\
 \hline
 \end{array}$$

Ex. 2. How long, after firing the Tower guns, may the report be heard at Shooter's Hill, supposing the distance to be 8 miles in a straight line?

$$\begin{array}{r}
 14 \\
 8 \\
 \hline
 3 \) \ 112 \\
 \hline
 \text{Ans. } 37\frac{2}{3} \text{ seconds.} \\
 \hline
 \end{array}$$

Ex. 3. After observing the firing of a large cannon at a distance, it was seven seconds before I heard the report; what was its distance? Ans. $1\frac{1}{2}$ mile.

Ex. 4. Perceiving a man at a distance hewing down a tree with an axe, I remarked that six of my pulsations passed between seeing him strike and hearing the report of the blow; what was the distance between us, allowing 70 pulses to a minute? Ans. 1 mile and 198 yards.

Ex. 5. How far off was the cloud, from which thunder issued, whose report was five pulsations after the flash of lightening; counting 75 to a minute? Ans. 1523 yards.

MISCELLANEOUS QUESTIONS.

Qu. 1. **W**HAT difference is there between a floor 28 feet long by 20 broad, and two others each of half the dimensions; and what do all three come to at 45s. per 100 square feet?

Ans, dif, 280 sq. feet, — Amount 18 guineas.
2. An

2. An elm plank is 14 feet 3 inches long, and I would have just a square yard slit off it; at what distance from the edge must the line be struck? Ans. $7\frac{29}{71}$ inches.

3. A ceiling contains 114 yards 6 feet of plastering, and the room 28 feet broad; what is the length of it? Ans. 36 $\frac{6}{7}$ feet.

4. A common joist is 7 inches deep, and $2\frac{1}{2}$ thick; but I want a scantling just as big again, that shall be 3 inches thick; what will the other dimension be? Ans. $11\frac{2}{3}$ inches.

5. A wooden trough cost me 3s 2d painting within, at 6d per yard; the length of it was 102 inches, and the depth 21 inches; what was the width? Ans. $27\frac{1}{4}$ inches.

6. If my court-yard be 47 feet 9 inches square, and I have laid a foot-path with Purbeck stone, of 4 feet wide, along one side of it; what will paving the rest with flints come to, at 6d per square yard? Ans. £5 16 0 $\frac{1}{4}$.

7. A ladder, 40 feet long, may be so planted, that it shall reach a window 33 feet from the ground on one side of the street; and, by only turning it over, without moving the foot out of its place, it will do the same by a window 21 feet high on the other side; what is the breadth of the street? Ans. 56 feet $7\frac{3}{4}$ inches.

8. The paving of a triangular court, at 18d per foot, came to 100l; the longest of the three sides was 88 feet; required the sum of the other two equal sides. Ans. 106.85 feet.

9. There are two columns in the ruins of Persepolis left standing upright: the one is 64 feet above the plain, and the other 50: in a straight line between these, stands an ancient small statue, the head of which is 97 feet from the summit of the higher, and 85 feet from the top of the lower column, the base of which measures just 76 feet to the centre of the figure's base. Required the distance between the tops of the two columns? Ans. 157 feet nearly.

10. The perambulator, or surveying wheel, is so contrived

trived, as to turn just twice in the length of a pole, or $16\frac{1}{2}$ feet; required the diameter? Ans. $2\cdot62\frac{1}{2}$ feet.

11. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer-wheel made two turns while the inner made but one: the wheels were both 4 feet high; and, supposing them fixed at the statutable distance of 5 feet asunder on the axle-tree, what was the circumference of the track described by the outer wheel? Ans. 63 feet nearly.

12. What is the side of that equilateral triangle whose area cost as much paving, at 8d a-foot, as the pallisading the three sides did, at a guinea a-yard? Ans. $72\cdot746$ feet.

13. In the trapezium ABCD are given, $AB = 13$, $BC = 31\frac{1}{3}$, $CD = 24$, and $DA = 18$, also B a right angle; required the area? Ans. $410\cdot122$.

14. A roof, which is 24 feet 8 inches by 14 feet 6 inches, is to be covered with lead at 8lb to the square foot: what will it come to at 18s per cwt?

Ans. $\text{£}22\ 19\ 10\frac{1}{4}$.

15. Having a rectangular marble slab, 58 inches by 27, I would have a square cut off parallel to the shorter edge; I would then have the like quantity divided from the remainder, parallel to the longer side; and this alternately repeated, till there shall not be the quantity of a foot left: what will be the dimensions of the remaining piece?

Ans. 20·7 inches by 6·086.

16. Given two sides of an obtuse-angled triangle which are 20 and 40 poles; required the third side, that the triangle may contain just an acre of land?

Ans. 58·876 or 23·099.

17. The end wall of a house is 24 feet 6 inches in breadth, and 40 feet to the eaves; $\frac{1}{3}$ of which is 2 bricks thick, $\frac{1}{3}$ more is $1\frac{1}{2}$ brick thick, and the rest 1 brick thick. Now the triangular gable rises 35 courses of bricks, 4 of which usually make a foot in depth, and this is but $4\frac{1}{2}$ inches, or half a brick thick: what will this piece of work come to at 5l. 10s per statute rod?

Ans. $\text{£}20\ 11\ 7\frac{1}{2}$.

18. 1f

18. If from a right-angled triangle, whose base is 12, and perpendicular 16 feet, a line be drawn parallel to the perpendicular, cutting off a triangle whose area is 24 square feet; required the sides of this triangle?

Ans. 6, 8, and 10.

19. The ellipse in Grosvenor-square measures 840 links across the longest way, and 612 the shortest, within the rails: now the walls being 14 inches thick, what ground do they inclose, and what do they stand upon?

Ans. $\left\{ \begin{array}{l} \text{inclose } 4 \text{ ac } 0 \text{ r } 6 \text{ p} \\ \text{stand on } 1760\frac{1}{2} \text{ sq. feet.} \end{array} \right.$

20. If a round pillar, 7 inches over, has 4 feet of stone in it; of what diameter is the column, of equal length, that contains 10 times as much?

Ans. 22.136 inches.

21. A circular fish-pond is to be made in a garden, that shall take up just half an acre; what must be the length of the cord that strikes the circle?

Ans. $27\frac{3}{4}$ yards.

22. When a roof is of a true pitch, the rafters are $\frac{1}{4}$ of the breadth of the building: now supposing the eaves-boards to project 10 inches on a side, what will the new ripping a house cost, that measures 32 feet 9 inches long, by 22 feet 9 inches broad on the flat, at 15s per square?

Ans. £8 15 9 $\frac{1}{2}$.

23. A cable, which is 3 feet long, and 9 inches in compass, weighs 22lb: what will a fathom of that cable weigh, which measures a foot about?

Ans. £78 $\frac{1}{2}$ lb.

24. My plumber has put 28 lb per square foot into a cistern 74 inches and twice the thickness of the lead long, 26 inches broad, and 40 deep; he has also put three stays across it within, 16 inches deep, of the same strength, and reckons 22s per cwt, for work and materials. I, being a mason, have paved him a workshop, 22 feet 10 inches broad, with Purbeck stone, at 7d per foot

foot; and upon the balance I find there is 35 6d due to him. What was the length of the workshop?

Ans. 32f 0 $\frac{1}{2}$ inches.

25. The distance of the centres of two circles, whose diameters are each 50, being given equal to 30; what is the area of the space inclosed by their circumferences?

Ans. 559.119.

26. If 20 feet of iron railing weigh half a ton, when the bars are an inch and a quarter square, what will 50 feet come to, at 3 $\frac{1}{2}$ d per lb, the bars being but $\frac{7}{8}$ of an inch square?

Ans. £20 0 2.

27. The area of an equilateral triangle, whose base falls on the diameter, and its vertex in the middle of the arc of a semicircle; is equal to 100: what is the diameter of the semicircle?

Ans. 26.32148.

28. It is required to find the thickness of the lead in a pipe, of an inch and quarter bore, which weighs 14lb per yard in length; the cubic foot of lead weighing 1132 $\frac{1}{2}$ ounces.

Ans. .20737 inches.

29. Supposing the expence of paving a semicircular plot, at 2s 4d per foot, come to 10l. what is the diameter of it?

Ans. 14.7737.

30. What is the length of a chord, which cuts off $\frac{1}{3}$ of the area, from a circle whose diameter is 289?

Ans. 278.6716.

31. My plumber has set me up a cistern, and his shop-book being burnt, he has no means of bringing in the charge, and I do not choose to take it down, to have it weighed; but by measure he finds it contains 64 $\frac{3}{10}$ square feet, and that it is precisely $\frac{1}{8}$ of an inch in thickness. Lead was then wrought at 21l per fother of 19 $\frac{1}{2}$ cwt. It is required from these items to make out the bill, allowing 6 $\frac{1}{2}$ oz for the weight of a cubic inch of lead?

Ans. £4 11 2.

32. What will the diameter of a globe be, when the solidity and superficial content are expressed by the same number?

Ans. 6.

33. A sack, that would hold 3 bushels of corn, is 22 $\frac{1}{2}$ inches

inches broad when empty; what will that sack contain which, being of the same length, has twice its breadth or circumference?

Anf. 12 bushels.

34. A carpenter is to put an oaken curb to a round well, at 8d per foot square: the breadth of the curb is to be $7\frac{1}{2}$ inches, and the diameter within $3\frac{1}{2}$ feet: what will be the expence?

Anf. 5s $2\frac{1}{4}$ d.

35. A gentleman has a garden 100 feet long, and 80 feet broad; now a gravel walk is to be made of an equal width half round it: what must the breadth of the walk be, to take up just half the ground?

Anf. 25.968 feet.

36. A may-pole whose top, being broken off by a blast of wind, struck the ground at 15 feet distance from the foot of the pole; what was the height of the whole may-pole, supposing the length of the broken piece to be 39 feet?

Anf. 75 feet.

37. Seven men bought a grinding stone, of 60 inches diameter, each paying $\frac{1}{7}$ part of the expence; what part of the diameter must each grind down for his share?

Anf. the 1st 4.4508, 2d 4.8400, 3d 5.3535,
4th 6.0765, 5th 7.2079, 6th 9.3935, 7th 22.6778.

38. A maltster has a kiln, which is 16 feet 6 inches square: but he wants to pull it down, and build a new one, that may dry three times as much at once as the old one; what must be the length of its side?

Anf. 28 feet 7 inches.

39. How many 3 inch cubes may be cut out of a 12 inch cube?

Anf. 64.

40. How long must be the tether of a horse, that will allow him to graze, quite around, just an acre of ground?

Anf. $39\frac{1}{4}$ yards.

41. What will the painting of a conical spire come to, at 8d per yard; supposing the height to be 118 feet, and the circumference of the base 64 feet?

Anf. £14 0 3 $\frac{1}{2}$.

42. The diameter of a standard corn bushel is $18\frac{1}{2}$ inches,

inches, and its depth 8 inches; what must the diameter of that bushel be, whose depth is $7\frac{1}{2}$ inches?

Anf. 19.1067.

43. Suppose the ball on the top of St. Paul's church is 6 feet in diameter; what did the gilding of it cost, at $3\frac{1}{2}$ d per square inch?

Anf. £237 10 1.

44. What will a frustum of a marble cone come to, at 12s per solid foot: the diameter of the greater end being 4 feet, that of the less end $1\frac{1}{2}$, and the length of the slant side 8 feet?

Anf. £30 1 10 $\frac{1}{4}$.

45. To divide a cone into three equal parts by sections parallel to the base, and to find the altitudes of the three parts, the height of the whole cone being 20 inches?

Anf. the upper part 13.867

the middle part 3.604

the lower part 2.528

46. A gentleman has a bowling-green, 300 feet long, and 200 feet broad, which he would raise 1 foot higher, by means of the earth to be dug out of a ditch that goes round it: to what depth must the ditch be dug, supposing its breadth to be every where 8 feet?

Anf. $7\frac{3}{8}$ feet.

47. How high above the earth must a person be raised, that he may see $\frac{1}{3}$ of its surface?

Anf. to the height of the earth's diameter.

48. A cubic foot of brass is to be drawn into a wire of $\frac{1}{16}$ of an inch in diameter; what will the length of the wire be, allowing no loss in the metal?

Anf. 97784.797 yards, or 55 miles 984.797 yards.

49. Of what diameter must the bore of a cannon be, which is cast for a ball of 24lb weight, so that the diameter of the bore may be $\frac{1}{16}$ of an inch more than that of the ball, and supposing a 9lb ball to measure 4 inches in diameter?

Anf. 5.757 inches.

50. Supposing the diameter of an iron 9lb ball to be 4 inches, as it is very nearly; it is required to find the diameters of the several balls weighing 1, 2, 3, 4, 6, 9, 12, 18, 24, 36, and 42 lb. and the caliber of their guns;

allowing

allowing $\frac{1}{30}$ of the caliber, or $\frac{1}{30}$ of the ball's diameter for windage,

Answer,

Wt ball	Diameter ball	Caliber gun
1	1.9230	1.9622
2	2.4228	2.4723
3	2.7734	2.8301
4	3.0526	3.1149
6	3.4943	3.5656
9	4.0000	4.0816
12	4.4026	4.4924
18	5.0397	5.1425
24	5.5469	5.6601
36	6.3496	6.4792
42	6.6844	6.8208

51. Supposing the windage of all mortars be allowed to be $\frac{1}{80}$ of the caliber, and the diameter of the hollow part of the shell to be $\frac{7}{10}$ of the caliber of the mortar ; It is required to determine the diameter and weight of the shell, and the quantity or weight of powder requisite to fill it, for each of the several sorts of mortars, namely, the 13, 10, 8, 5.8, and 4.6 inch mortar ?

Answer.

Calib. mort.	Diameter shell	Wt. shell empty	Wt. of powder	Wt. shell filled
4.6	4.523	8.320	0.583	8.903
5.8	5.703	16.677	1.168	17.845
8	7.867	43.764	3.065	46.829
10	9.833	85.470	5.986	91.462
13	12.783	187.791	13.151	200.942

52. How many shot are in a complete square pile, each side of the base containing 29 ?

Ans. 8555.

53. How

53. How many shot are in a complete oblong pile, the length of the base containing 49, and the breadth 19?

Ans. 8170.

54. How many shot are in a triangular pile, each side of the base being 50?

Ans. 22100.

55. How many shot are in an unfinished triangular pile, the side of the bottom being 50, and top 20?

Ans. 20770.

56. How many shot are in an unfinished oblong pile, having the corner row 12, and the sides of the top 40 and 10?

Ans. 8606.

57. If a heavy sphere, whose diameter is 4 inches, be let fall into a conical glass, full of water, whose diameter is 5, and altitude 6 inches; it is required to determine how much water will run over?

Ans. 26.272 cubic inches, or near $\frac{35}{47}$ parts of a pint.

58. The dimensions of the sphere and cone being the same as in the last question, and the cone only $\frac{1}{3}$ full of water; required what part of the axis of the sphere is immersed in the water?

Ans. .546 parts of an inch.

59. The cone being still the same, and $\frac{1}{3}$ full of water; required the diameter of a sphere that may be just all covered by the water?

Ans. 2.445996.

60. If I see the flash of a cannon, fired by a ship in distress at sea, and hear the report 33 seconds after, how far is she off?

Ans. 7 $\frac{1}{14}$ miles.

61. Being one day ordered to observe how far a battery of cannon was from me, I counted by my watch 17 seconds between the time of seeing the flash and hearing the report; how far was the battery from me?

Ans. 3 $\frac{1}{2}$ miles.

62. An irregular piece of lead ore weighs in air 12 ounces, but in water only 7; and another fragment weighs in air 14 $\frac{1}{2}$ ounces, but in water only 9; required their comparative densities?

Ans. as 145 to 132.

63. Supposing the cubic inch of common glass weigh 1.36 ounces troy, the same of salt water .5427, and of brandy .48926; then a seaman having a gallon of that liquor in a glass bottle, which weighs 3 $\frac{1}{2}$ lb troy out of

water, and to conceal it from the officers of the customs, throws it overboard. It is required to determine, if it will sink, how much force will just buoy it up?

Ans. 12 8968 ounces.

64. Suppose by measurement it be found that a ship of war, with its ordnance, rigging and appointments, draws so much water as to displace 50000 cubic feet of water; required the weight of the vessel?

Ans. 1395 $\frac{1}{10}$ tons.

A T A B L E

OF THE

AREAS of the SEGMENTS of a CIRCLE,

Whose Diameter is Unity, and supposed to be divided
into 1000 equal Parts.

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
·001	·000042	·027	·005867	·053	·016007
·002	·000119	·028	·006194	·054	·016457
·003	·000219	·029	·006527	·055	·016911
·004	·000337	·030	·006865	·056	·017369
·005	·000470	·031	·007209	·057	·017831
·006	·000618	·032	·007558	·058	·018296
·007	·000779	·033	·007913	·059	·018766
·008	·000951	·034	·008273	·060	·019239
·009	·001135	·035	·008638	·061	·019716
·010	·001329	·036	·009008	·062	·020196
·011	·001533	·037	·009383	·063	·020680
·012	·001746	·038	·009763	·064	·021168
·013	·001968	·039	·010148	·065	·021659
·014	·002199	·040	·010537	·066	·022154
·015	·002438	·041	·010931	·067	·022652
·016	·002685	·042	·011330	·068	·023154
·017	·002940	·043	·011734	·069	·023659
·018	·003202	·044	·012142	·070	·024168
·019	·003471	·045	·012554	·071	·024680
·020	·003748	·046	·012971	·072	·025195
·021	·004031	·047	·013392	·073	·025714
·022	·004322	·048	·013818	·074	·026236
·023	·004618	·049	·014247	·075	·026761
·024	·004921	·050	·014681	·076	·027289
·025	·005230	·051	·015119	·077	·027821
·026	·005546	·052	·015561	·078	·028356

AREAS OF THE SEGMENTS OF A CIRCLE. 317

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
·079	·028894	·114	·049528	·149	·073161
·080	·029435	·115	·050165	·150	·073874
·081	·029979	·116	·050804	·151	·074589
·082	·030526	·117	·051446	·152	·075306
·083	·031076	·118	·052090	·153	·076026
·084	·031629	·119	·052736	·154	·076747
·085	·032186	·120	·053385	·155	·077469
·086	·032745	·121	·054036	·156	·078194
·087	·033307	·122	·054689	·157	·078921
·088	·033872	·123	·055345	·158	·079649
·089	·034441	·124	·056003	·159	·080380
·090	·035011	·125	·056663	·160	·081112
·091	·035585	·126	·057326	·161	·081846
·092	·036162	·127	·057991	·162	·082582
·093	·036741	·128	·058658	·163	·083320
·094	·037323	·129	·059327	·164	·084059
·095	·037909	·130	·059999	·165	·084801
·096	·038496	·131	·060672	·166	·085544
·097	·039087	·132	·061348	·167	·086289
·098	·039680	·133	·062026	·168	·087036
·099	·040276	·134	·062707	·169	·087785
·100	·040875	·135	·063389	·170	·088535
·101	·041476	·136	·064074	·171	·089287
·102	·042080	·137	·064760	·172	·090041
·103	·042687	·138	·065449	·173	·090797
·104	·043296	·139	·066140	·174	·091554
·105	·043908	·140	·066833	·175	·092313
·106	·044522	·141	·067528	·176	·093074
·107	·045139	·142	·068225	·177	·093836
·108	·045759	·143	·068924	·178	·094601
·109	·046381	·144	·069625	·179	·095366
·110	·047005	·145	·070328	·180	·096134
·111	·047632	·146	·071033	·181	·096903
·112	·048262	·147	·071741	·182	·097674
·113	·048894	·148	·072450	·183	·098447

Height	Area Deg.	Height	Area Deg.	Height	Area Deg.
184	099221	219	127285	254	157019
185	099997	220	128113	255	157890
186	100774	221	128942	256	158762
187	101553	222	129773	257	159636
188	102334	223	130605	258	160510
189	103116	224	131438	259	161386
190	103900	225	132272	260	162263
191	104685	226	133108	261	163140
192	105472	227	133945	262	164019
193	106261	228	134784	263	164899
194	107051	229	135624	264	165780
195	107842	230	136465	265	166663
196	108636	231	137307	266	167546
197	109430	232	138150	267	168430
198	110226	233	138995	268	169315
199	111024	234	139841	269	170202
200	111823	235	140688	270	171089
201	112624	236	141537	271	171978
202	113426	237	142387	272	172867
203	114230	238	143238	273	173758
204	115035	239	144091	274	174649
205	115842	240	144944	275	175542
206	116650	241	145799	276	176435
207	117460	242	146655	277	177330
208	118271	243	147512	278	178225
209	119083	244	148371	279	179122
210	119897	245	149230	280	180019
211	120712	246	150091	281	180918
212	121529	247	150953	282	181817
213	122347	248	151816	283	182718
214	123167	249	152680	284	183619
215	123988	250	153546	285	184521
216	124810	251	154412	286	185425
217	125634	252	155280	287	186329
218	126459	253	156149	288	187234

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
289	188140	324	220404	359	253590
290	189047	325	221340	360	254550
291	189955	326	222277	361	255510
292	190864	327	223215	362	256471
293	191775	328	224154	363	257433
294	192684	329	225093	364	258395
295	193596	330	226033	365	259357
296	194509	331	226974	366	260320
297	195422	332	227915	367	261284
298	196337	333	228858	368	262248
299	197252	334	229801	369	263213
300	198168	335	230745	370	264178
301	199085	336	231689	371	265144
302	200003	337	232634	372	266111
303	200922	338	233580	373	267078
304	201841	339	234526	374	268045
305	202761	340	235473	375	269013
306	203683	341	236421	376	269982
307	204605	342	237369	377	270951
308	205527	343	238318	378	271920
309	206451	344	239268	379	272890
310	207376	345	240218	380	273861
311	208301	346	241169	381	274832
312	209227	347	242121	382	275803
313	210154	348	243074	383	276775
314	211082	349	244026	384	277748
315	212011	350	244980	385	278721
316	212940	351	245934	386	279694
317	213871	352	246889	387	280668
318	214802	353	247845	388	281642
319	215733	354	248801	389	282617
320	216666	355	249757	390	283592
321	217599	356	250715	391	284568
322	218533	357	251673	392	285544
323	219468	358	252631	393	286521

320 AREAS OF THE SEGMENTS OF A CIRCLE.

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
394	287498	430	322928	466	358725
395	288476	431	323918	467	359723
396	289453	432	324909	468	360721
397	290432	433	325900	469	361719
398	291411	434	326892	470	362717
399	292300	435	327882	471	363715
400	293369	436	328874	472	364713
401	294349	437	329866	473	365712
402	295330	438	330858	474	366710
403	296311	439	331850	475	367709
404	297292	440	332843	476	368708
405	298273	441	333836	477	369707
406	299255	442	334829	478	370706
407	300238	443	335822	479	371705
408	301220	444	336816	480	372704
409	302203	445	337810	481	373703
410	303187	446	338804	482	374702
411	304171	447	339798	483	375702
412	305155	448	340793	484	376702
413	306140	449	341787	485	377701
414	307125	450	342782	486	378701
415	308110	451	343777	487	379700
416	309095	452	344772	488	380700
417	310081	453	345768	489	381699
418	311068	454	346764	490	382699
419	312054	455	347759	491	383699
420	313041	456	348755	492	384699
421	314029	457	349752	493	385699
422	315016	458	350748	494	386699
423	316004	459	351745	495	387699
424	316992	460	352742	496	388699
425	317981	461	353739	497	389699
426	318970	462	354736	498	390699
427	319959	463	355732	499	391699
428	320948	464	356730	500	392699
429	321938	465	357727		

THE USE OF THE TABLE.

In the foregoing table, each number in the column of *area seg.* is the area of the circular segment whose height, or the versed sine of its half arc, is the number immediately on the left of it, in the column of *heights*; the diameter of the circle being 1, and its whole area .785398.

The use of this table is to find, by it, the area of the segment of any other circle, whatever be the diameter. And this is done by first dividing the height of any proposed segment by its own diameter, and the quotient is a decimal to be sought in the column of heights, and against it is the tabular area to be taken out, which is similar to the proposed segment. Then this tabular area, being multiplied by the square of the given diameter, will be the area of the segment required; because similar areas are to each other as the squares of their diameters.

EXAMPLE.

So if it be required to find the area of a segment of a circle, whose height is $3\frac{1}{4}$, the diameter being 50.

Here $50 \div 3\frac{1}{4} = .065$ quot. or tabular height,
and the tab. seg. is .021659
which multiplied by 2500 the square of the diam.

gives

54.147500 the area required.

But,

But, in dividing the given height by the diameter, if the quotient do not terminate in three places of decimals without a fractional remainder, then the area for that fractional part ought to be proportioned for, thus: Having found the tabular area answering to the first three decimals of the quotient, take the difference between it and the next following tabular area, which difference multiply by the fractional remaining part of the quotient, and the product will be the corresponding proportional part, to be added to the first tabular area.

So if the height of a proposed segment were $3\frac{1}{2}$, to the diameter 50.

Here 50)	$3\frac{1}{2}$	($\cdot 066\frac{2}{3}$	Then
to	$\cdot 066$	answers	$\cdot 022154$	
the next area is			$\cdot 022652$	
				<hr/>
their difference is			498	
$\frac{2}{3}$ of which is			232	
which added to			$\cdot 022154$	
				<hr/>
gives the whole tab. area			$\cdot 022386$	
And this multiplied by			2500	
				<hr/>
gives the area			$55\cdot 965000$	sought

FINIS.

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